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Verification of Forecast Effectiveness for Selected Volatility Estimators

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Abstract

Aim: The aim of this study was to determine which volatility forecasting method produces results that are closest to the actual and whether the use of estimators with OHLC prices affects forecast accuracy.

Methodology: This study examined five models – a historical model, GARCH(1,1) and three GARCH models with selected volatility estimators (Parkinson, Garman-Klass and Rogers-Satchell). The sample used daily prices, with each instrument having 2001 observations and a 20-day forecast horizon. Forecast accuracy was assessed using RMSE and MAE.

Findings: The empirical results determined that no specific approach is universally regarded as superior. It is recommended that naive methods or the standard GARCH method be used as they are simpler than the complex models with selected estimators and save operating time. Volatility estimators enhanced accuracy for stocks but not for other instruments. For stocks, estimator-based models obtained better results; for others, classical methods were more effective.

Implications and recommendations: This study can assist researchers in selecting the appropriate model for specific data and indicate whether the use of a different estimator would enrich the results of forecasts. Further research could investigate the impact of higher frequency data on the performance of volatility estimators.

Originality/value: The study examined whether the Polish market responds to volatility estimators similarly to global markets. It also confirmed that the best model varies by instrument: the model with Rogers-Satchell estimator for stocks, GARCH(1,1) for currencies, and the historical method for commodities.

Keywords: volatility, volatility forecasting, volatility estimators, OHLC prices

1. Introduction

Risk and volatility are extremely important concepts in modern finance. There is increasing recognition of the need to manage risk in order to limit losses to a level acceptable to the individual (Jajuga, 2007; Jajuga & Jajuga, 2007). Measuring and forecasting volatility is part of the risk management process, more specifically market risk. Such forecasts can have a number of applications in finance, e.g. in the selection of instruments for portfolio construction and in option pricing.

There is a trend towards more precise models, allowing for increasingly accurate forecasts (Hsu et al., 2014; Fiszeder, 2020). Often only closing prices are used in studies, but opening, high, and low prices are also available and can enrich the analysis, leading to better results. A solution to this situation is to use volatility estimators which allow all prices available on a given day to be taken into account.

In the literature there are studies of volatility estimators using various models (GARCH, HAR-RV-X) usually on stock prices from around the world. Different countries have been shown to respond differently to volatility estimators, but in general the estimators improve forecast performance; therefore, this study employed data from the Polish market and examined not only stocks, but also currencies and commodities. This approach allowed the author to verify whether the results were the same despite different data, and if the volatility estimators consistently improve the results.

The aim of the study was to determine which volatility forecasting method produces results that are closest to the actual results, and whether the use of estimators using both closing and opening prices, as well as minimum and maximum prices affects the accuracy of the forecasts. The study used naive models and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models extended with selected estimators.

This article examines five models – a historical model, GARCH(1,1) and three GARCH models with selected volatility estimators, including Parkinson, Garman-Klass and Rogers-Satchell, to verify whether the use of daily OHLC prices (Open-High-Low-Close prices) on selected financial instruments improves volatility forecasts. This study also used the daily prices of stocks, currency pairs and commodities, and the empirical results were not consistent across all instruments. For stock prices, volatility estimators improved the forecasts, however for commodities and currencies the standard GARCH and historical models ranked higher in accuracy.

2. Literature Review

Volatility is a very significant element in financial markets. Modelling and forecasting of volatility has recently become increasingly important, finding many applications in finance. Forecasting volatility is also part of the risk management process, integral to financial markets. In response to this need, a number of new methods were developed, starting with the ARCH model proposed by Engle (1982) and the GARCH model by Bollerslev (1986).

The next step was model enhancement, which aimed for the best possible fit of model parameters to reality. One of the solutions was to build estimators based on the opening, highest and lowest daily prices, and not merely focusing on the closing price. Parkinson's (1980) estimator, based on the high--low price range, proved to deliver better results than an estimator based on closing prices. Many more followed, resulting in a range of volatility estimators based on OHLC prices (Fiszeder, 2020).

Thus the research question arises – do volatility estimators, specifically Parkinson's, Garman-Klass and Rogers-Satchell, improve forecasts and, if so, which one produces an outcome closest to actual realised volatility?

Hsu et al. (2014) tested this theory, constructing a study based on Nasdaq-100 stock index returns. The research included a GARCH(1,1) model enhanced by various volatility estimators, later evaluated by popular error measures (MAE, MSE, RMSE and LL). The applied estimators were overnight volatility, Parkinson, Garman-Klass, Rogers-Satchell, RV, RBP and VIX, leading to the conclusion that the estimators did improve the model forecasts, except the overnight volatility estimator.

Korkusuz et al. (2023) proposed a different approach, conducting their study within a HAR-RV-X (Heterogeneous Autoregressive model of Realised Volatility with eXogenous variables) framework instead of GARCH, yet the aim was the same, to test whether volatility estimators improve forecasts. The study included six estimators – Parkinson, Garman-Klass, Rogers-Satchell, Yang-Zhang, Close-to-Close and overnight volatility. Data chosen for this study consisted of the stock market indices of the G7 (Canada, France, Italy, Germany, Japan, the UK and the USA) between 2009 and 2021. The results did not identify a definitive best estimator for every market, but it could be seen that the simplest of them (the overnight volatility) improved the results in all but one market. The scores for estimators varied between markets, but they did indeed improve the results and should be included in volatility forecasts.

Another improvement to research was the use of high-frequency data with volatility estimators. Todorova & Husmann (2012) employed intraday data for 25 German stocks to analyse how Parkinson, Garman-Klass, Rogers-Satchell, Yang and Zhang and the realised range estimators compare. It was confirmed that the realised range (the sum of intraday ranges) performed better than daily estimators.

More recent studies combined GARCH-based framework and advanced machine learning models, sometimes with the addition of high-frequency data (Celestin et al., 2025; Chung, 2024). The addition of machine learning allowed one to reduce noise, enhance the models and overall improve forecast capability, but the GARCH models still play a critical role in volatility forecasting. The research recommend hybrid solutions, namely a GARCH model enhanced with machine learning techniques.

One can observe the more frequent use of artificial intelligence in volatility forecasting. Not only machine learning, but also deep learning (Tripathy et al., 2025) and neural networks were seen in recent studies (Bahoo et al., 2024; Ge et al., 2022), with the results being usually more effective; however, there are still areas that require further research as the models can be difficult to compare.

Some researchers instead of introducing new estimators, decided to use range-based models such as CARR, which include high and low prices in any time interval, not used in the standard GARCH model. Ng et al. (2017) showed in their study that CARR models are an efficient method for forecasting volatility and can compete with a GARCH-based framework, yet GARCH methods are still the most commonly used for volatility forecasting because of their simplicity and possibilities for enhancement.

The newer methods provide a broader background, but this study focused on the classical estimators and time series methodology, still commonly used to correctly describe volatility and to compete with more modern solutions, as shown in the literature review.

The author decided to use the Polish market to test whether Parkinson, Garman-Klass and Rogers--Satchell estimators improve the forecasts. The study was based on the daily prices of stocks, along with currency pairs and commodities, testing if each instrument would align in determining the best model and estimator. The aim was to address the existing research gap in the literature regarding the use of volatility estimators within the context of the Polish financial market. The best model was selected based on the lowest error value. The methodology and empirical results are shown further.

3. Methodology

3.1. Volatility Measurement

The concept of volatility was defined by J. C. Hull (1989) as follows:

“The volatility, σ, of a stock is a measure of our uncertainty about the returns provided by the stock.” (p. 342)

The classic estimator used to measure volatility is standard deviation. Sometimes variance is also encountered, but standard deviation is easier to interpret and therefore used as the classic estimator. The formula is as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(r_i-\bar{r})^2}{n-1}}, \tag{1}$$

where r is the logarithmic return calculated from closing prices, $\bar{r}$ is the sample mean and n is the sample size.

To test whether using not only the closing prices but also the opening, minimum and maximum prices during the day would change the accuracy of the forecast, three OHLC estimators were used. The formulae are quoted below.

The following symbols describe the quotations during a given day (t):

- O_t – opening price,
- H_t – highest price,
- L_t – lowest price,
- C_t – closing price.

According to research, these estimators improve the volatility predictions of the standard GARCH model (Hsu et al., 2014) tested in this article.

The first estimator was the Parkinson estimator, with the formula shown below:

$$\sigma_{Pt}^2 = \frac{(h_t-l_t)^2}{4\,ln(2)}, \tag{2}$$

where $h_t = \ln(H_t) - \ln(O_t)\,, l_t = \ln(L_t) - \ln\,(O_t)$.

This version of the estimator was proposed by P. Fiszeder and G. Perczak to control for overestimation of volatility when drift occurs. The Parkinson estimator is one of the most popular variance estimators due to its simplicity and efficiency (Fiszeder, 2020).

The next proposed estimator was Garman-Klass calculated as shown below:

$$\sigma_{GKt}^2 = \frac{1}{2}(h_t - l_t)^2 - (2\,ln(2) - 1)c_t^2, \tag{3}$$

where $c_t = \ln(C_t) - \ln(O_t)$.

This formula is also intended to control for overestimation when drift occurs. It is a simplified form of the variance estimator, but the analytical formula is only slightly more accurate, hence the simplified formula is recommended (Fiszeder, 2020).

The final estimator was Rogers-Satchell, using the following equation:

$$\sigma_{RSt}^2 = h_t(h_t - c_t) + l_t(l_t - c_t). \tag{4}$$

This estimator takes into account a drift different from zero, but is able to take a zero value under certain conditions, namely when $O_t = L_t$ and $C_t = H_t$ or $O_t = H_t$ and $C_t = L_t$, because such a relationship is described by the drift phenomenon (Fiszeder, 2020).

3.2. Historical Method

Volatility forecasts can be determined as an appropriately modified volatility, based on available historical data, or implied volatility, calculated using the prices of an option written on a financial instrument (Piontek, 2002). This study was conducted using methods that apply historical volatility to estimate forecasts. The first model, called the historical method, is shown below.

The forecast was determined from the realised variance, using the Random Walk Model, where (f, t) symbolises the forecast for the given day, while (h, t) represents the historical value for the day before:

$$\sigma_{f,t}^2(RW) = \sigma_{h,t-1}^2. \tag{5}$$

The following formula was then used to obtain a volatility forecast:

$$\sigma_{f,t} = \sqrt{\sigma_{f,t}^2}\,. \tag{6}$$

This is one of the simplest methods of forecasting volatility, which allows for the results produced to be compared with those of more complex methods (Piontek, 2002).

3.3. The GARCH Model

The standard version of the GARCH model, i.e. GARCH(1,1), was chosen as the second method of analysis. The study was conducted using R and the rugarch package, which is one of the recommended packages for estimating GARCH models (Hill & McCullough, 2019). In most cases, GARCH(p,q) models where $p \leq 2$ and $q \leq 2$ are sufficient models, being flexible and simple to use and not losing important economic information (Knight & Satchell, 2011). The model can be represented by the following formula (Ghalanos, 2022):

$$\sigma_t^2 = \omega + \sum_{j=1}^{m} \zeta_j v_{t-j} + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{7}$$

where σ_t^2 is the conditional variance, ε_t^2 is the residual from the mean filtering process and v_t are the added additional OHLC variance estimators that were described in an earlier subsection. The model adds the estimators with a lag (Ghalanos, 2022), and it is based on the GARCH model introduced by Bollerslev (1986). The study adopted a Student's t-distribution, assuming that market returns tend to have fatter tails than a normal distribution.

3.4. Error Measurement

To calculate prediction errors, two measures were applied which are often used in the literature to assess volatility forecasts (Piontek, 2002; Doman & Doman, 2009; Knight & Satchell, 2011; Hsu et al., 2014). The forecast errors were calculated for a set of twenty forecasts and the variability realised on the days in question. The equations are presented in (8) and (9):

- RMSE – Root Mean Squared Error,

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\sigma_{f,t} - \sigma_{h,t}\right)^2}. \tag{8}$$

- MAE – Mean Absolute Error.

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\sigma_{f,t} - \sigma_{h,t}\right|. \tag{9}$$

Both of these measures are symmetric, but the RMSE measure accounts more strongly for large individual forecast errors, whereas the MAE measure – more weakly (Piontek, 2002).

3.5. Sample and Data Collection

In the empirical research, five volatility forecasting methods were compared. The rates of three types of financial instruments were selected, namely stock prices of listed companies, rates of currency pairs and rates of commodity futures contracts. Five sample instruments were selected in each category: the stock prices of KGHM Polska Miedź SA, PKN ORLEN SA, CD Projekt Capital Group, Wielton SA and Budimex SA, the rates of USD/GBP, USD/EUR, USD/JPY, EUR/PLN and CAD/PLN currency pairs, and the rates of commodity futures contracts such as gold, copper, nickel, silver or platinum. The purpose of this variety of instruments was to see whether this would affect the subsequent choice of model, and if instruments from the same category would agree on the best model. The instruments were selected randomly to ensure objectivity, generalisability, and statistical validity, especially when testing financial models across diverse conditions. In this study, quotations with the US dollar (USD) as the base currency (e.g. USD/GBP, USD/EUR) were adopted. This choice was determined by both the research focus on analysing the USD's value and the format of the sourced data, which allowed for the preservation of their original characteristics and avoided the need for additional transformations.

All the calculations were performed on logarithmic rates of return, as volatility analysis is most often performed on such rates and their use is justified in various studies (Piontek, 1999; Doman & Doman, 2009). The logarithmic return is defined as follows:

$$r_t = ln\left(\frac{C_t}{C_{t-1}}\right). \tag{10}$$

A total of 2001 observations were used in the calculations, allowing to calculate 2000 returns. Forecasts were calculated for the next 20 days and compared with real data from that period. Actual daily volatility for the forecast period was proxied by the absolute value of daily log-return (realised volatility). The selection of an appropriate estimation window is crucial for volatility forecasting, balancing the need for sufficient data against the risk of incorporating outdated information due to structural changes in market dynamics. To ensure a robust comparison of model performance in this study, a consistent estimation period (2000 returns) and forecast horizon (20 days) were applied across all instruments. This approach provided a common basis for evaluating which model forecasted volatility most accurately.

All the forecasts were set for 20 days, with the same start date for all instruments (2.01.2023); however, because of different trading days, the observation period varied. For stock prices, this 2.01.2015−−30.12.2022, while for currency pairs 7.04 or 8.04.2015−30.12.2022 (depending on the pair), for copper 13.04.2015−30.12.2022, for nickel 28.01.2015−30.12.2022, for platinum 27.07.2016−−30.12.2022, for silver 13.03.2015−30.12.2022, and for gold 6.04.2015−30.12.2022. These periods were applied to obtain 2001 observations. Data were taken from stooq.pl and investing.pl.

The models described above were used for the forecasts, whilst volatility estimators based on OHLC prices were adopted as an extension of the standard GARCH model.

The calculations employed 2000 logarithmic rates, such that the oldest observation was discarded when the latest forecast was taken into account. This created the so-called rolling window, allowing the most recent observations to be included and comparing how the model and the estimator changed forecasts based on the new information. This technique enables model parameters to change over time.

4. Results

Each instrument underwent the same process using the methods previously indicated. To illustrate this, the analysis of the CD Projekt Capital Group and its detailed results are shown. The results for the remaining instruments are presented in Table 3.

4.1. CD Projekt Capital Group

Table 1 presents the results of the volatility forecasts for CD Projekt. The first column shows the historical method, whereas the following columns show the results obtained with the GARCH model with volatility estimators added. The last column shows the actual price change for the period, calculated from closing prices.

Table 1. Volatility predictions for CD Projekt

Date	Historical	GARCH	GARCH-P	GARCH-G-K	GARCH-R-S	Realised
02.01.2023	0.02829	0.02462	0.02208	0.02117	0.02086	0.00232
03.01.2023	0.02829	0.02284	0.02086	0.02034	0.02045	0.00263
04.01.2023	0.02829	0.02141	0.02141	0.02141	0.02040	0.01170
05.01.2023	0.02829	0.02104	0.02066	0.02013	0.01989	0.03892
09.01.2023	0.02830	0.02603	0.02604	0.02604	0.02521	0.00886
10.01.2023	0.02830	0.02419	0.02419	0.02480	0.02435	0.03251
…	…	…	…	…	…	…
19.01.2023	0.02834	0.02727	0.02704	0.02630	0.02584	0.00587
20.01.2023	0.02833	0.02518	0.02518	0.02518	0.02518	0.00015
23.01.2023	0.02833	0.02327	0.02319	0.02299	0.02254	0.02371
24.01.2023	0.02834	0.02391	0.02311	0.02250	0.02207	0.01320
25.01.2023	0.02834	0.02319	0.02287	0.02268	0.02221	0.01808
26.01.2023	0.02834	0.02338	0.02211	0.02192	0.02160	0.02359
27.01.2023	0.02835	0.02397	0.02218	0.02220	0.02201	0.00566
30.01.2023	0.02835	0.02253	0.02253	0.02253	0.02083	0.00825

Source: own research.

Figure 1 illustrates these results. The actual values are shown as actual volatility, i.e. as absolute values of the logarithmic rate of return.

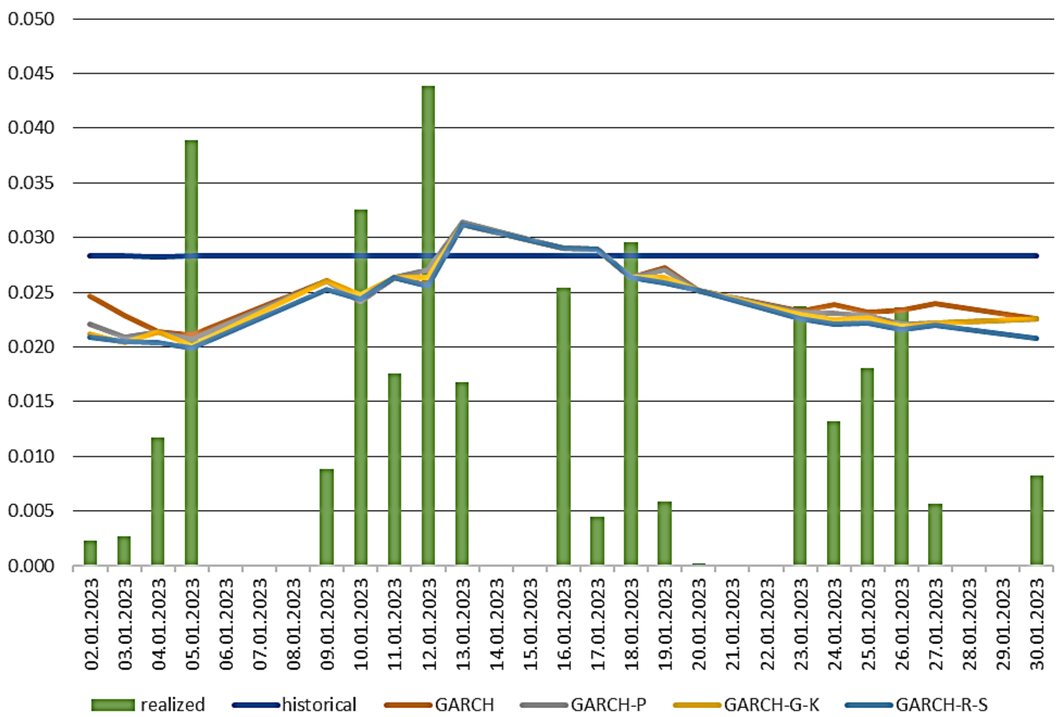

Fig. 1. Volatility predictions for CD Projekt

Source: own research.

The RMSE and MAE prediction errors were calculated for each model and are shown in Table 2.

Table 2. Prediction errors for CD Projekt

	Historical	GARCH	GARCH-P	GARCH-G-K	GARCH-R-S
RMSE	0.03412	0.03187	0.03170	0.03149	0.03116
MAE	0.02872	0.02682	0.02660	0.02642	0.02619

Source: own research.

In this case, the best model was the GARCH model with the Rogers-Satchell estimator, taking the lowest values in both error measures. The model with the Garman-Klass estimator was ranked second.

4.2. Final Results

Table 3 presents a summary of the most successful models.

Table 3. Summary of results

Instrument	Best model	Second best model
Stocks		
Budimex SA	GARCH R-S	GARCH P
CD Projekt Capital Group	GARCH R-S	GARCH G-K
KGHM Polska Miedź SA	Historical	GARCH P
PKN Orlen SA	GARCH G-K	GARCH(1,1)
Wielton SA	GARCH R-S	GARCH G-K
Currencies		
CAD/PLN	GARCH(1,1)	GARCH P
EUR/PLN	GARCH(1,1)	GARCH R-S
USD/EUR	Historical	(GARCH P)
USD/GBP	GARCH(1,1)	GARCH P
USD/JPY	Historical	(GARCH(1,1))
Commodities		
Copper	Historical	GARCH G-K
Nickel	GARCH(1,1)	GARCH R-S
Platinum	Historical	GARCH(1,1)
Silver	Historical	GARCH(1,1)
Gold	Historical	GARCH R-S

Source: own research.

As can be seen, the results differ between the selected instruments over the period. For stock returns, the best forecast model was the GARCH class model with the Rogers-Satchell estimator and the model with the Garman-Klass estimator. For currency returns, the standard GARCH model proved to be the best. The historical method proved to be best for the USD/EUR and USD/JPY exchange rates, performing significantly better than the other models. The GARCH model with the Parkinson estimator also performed well for this type of instrument. For commodity futures returns, the best method by far was to model volatility using the historical method. The standard GARCH(1,1) model also appears frequently for these instruments.

To obtain a measurable result the author assigned two points for the “best model” and one point for the “second best model”. It could then be noted that the overall highest scoring model was the historical method with 14 points, followed by GARCH(1,1) with 12, closely followed by the GARCH model with Rogers-Satchell estimator (nine points) and the models with Parkinson and Garman-Klass (5 points each).

In light of these considerations it may be advisable to consider adopting the more straightforward approaches represented by the historical and standard GARCH models. If only stocks were considered, then it could be argued that the Rogers-Satchell estimator should be used, as it was demonstrated that this method performed better in this particular context.

5. Discussion and Conclusion

Risk is inherent in financial markets and investments, and managing it has become crucial for reducing risk to an acceptable level. Volatility, often seen as uncertainty, plays a key role in risk measurement and forecasting. Accurate volatility models are essential for applications such as portfolio construction and option pricing. The aim of this study was to determine which volatility forecasting method produced results closest to the actual results, and whether the use of estimators using both closing and opening prices, minimum or maximum prices affected the accuracy of forecasts.

The methodology section outlines the tools for the study, detailing methods for measuring volatility and the models used. Prediction errors were examined to identify the most effective forecasts. In the next section, the study period and forecast duration were selected, employing 2000 logarithmic returns from stocks, currency pairs, and commodity futures, with volatility forecasts set for 20 days using a rolling window method. Five models were tested: Random Walk, GARCH(1,1), and GARCH models with three volatility estimators: Parkinson, Garman-Klass, and Rogers-Satchell. The study assumed a Student's *t*-distribution for market returns. The results, including prediction errors calculated by RMSE and MAE, are shown and compared across instruments to identify the best models.

For stock returns, the best forecast model was the GARCH class model with the Rogers-Satchell estimator and the model with the Garman-Klass estimator. For exchange rate returns, the standard GARCH(1,1) model was best, followed by the historical method, whilst for futures, the historical method proved to be the best, followed by the classical GARCH(1,1) model.

However, the models produced such similar results that it proved difficult to clearly identify the best model for all instruments. After counting the scores for each method, it could be seen that the historical method and the standard GARCH model were the overall best models, despite the assumption that the additional estimators would improve prediction performance (as confirmed by studies on data from other countries (Hsu et al., 2014; Korkusuz et al., 2023)). These findings could have been affected by the use of different classes of instruments, whereas researchers usually employ just stock prices. If only stock results would be considered, the GARCH with Rogers-Satchell estimator were the best, followed by Garman-Klass. Perhaps stocks have more intraday relative volatility, captured by the estimators as opposed to the other instruments that might have the daily range information already reflected by the closing price. This area can be further studied as to why those instruments react differently.

Future research could build upon this study and include higher frequency data to see if intraday data on the Polish market would show that volatility estimators improve the results of all instrument types. Another approach that could improve this study would be to test the significance of differences as this could formally prove whether one method is better than others (e.g. the Diebold-Mariano test). Researchers could also include a different forecast horizon (other than 20 days), as the obtained results related to a specific period of time and may have depended on market conditions at that time. While it is agreed that volatility estimators improve forecasts, Patton (2011), Hansen & Lunde (2005) with Hung et al. (2013) suggested that the use of imperfect volatility proxies in combination with loss functions that are not robust might lead to incorrect conclusions and rankings; applying the MSE or

QLIKE loss functions or considering SPA methodology might have been more justified. This article did not address these additions due to limitations in data availability and volume of work, but it could be an area for further studies.

In conclusion, it is recommended that a prediction model should be selected based on simplicity or on the approach requiring the least amount of data as these proved to obtain the best scoring in general. The only situation where volatility estimators should be considered is when analysing stock prices, as they improved the forecasts results for this instrument. This information could be helpful for future research, depending on what instrument type is being analysed, as choosing an inappropriate estimator could lead to underestimating or overestimating market risk. An appropriate estimator and thus an improved volatility forecast precision, contribute directly to more accurate Value-at-Risk (VaR) estimation, more efficient capital allocation, and more reliable option pricing strategies. Overly complicated models may overestimate volatility, potentially leading to excessively high option premiums and mispriced financial instruments. Historical models, while simpler, can offer more stable and conservative VaR estimates at lower computational cost. Consequently, in portfolio construction and risk management, simpler models may help investors avoid risk overestimation and overdiversification, thereby improving capital efficiency.

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Weryfikacja skuteczności prognoz dla wybranych estymatorów zmienności

Streszczenie

Cel: Celem niniejszego badania jest określenie, która metoda prognozowania zmienności daje wyniki najbardziej zbliżone do rzeczywistych i czy zastosowanie estymatorów wykorzystujących ceny OHLC wpłynie na dokładność prognoz.

Metodyka: W niniejszym badaniu przeanalizowano pięć modeli: model historyczny, GARCH(1,1) i trzy modele GARCH z wybranymi estymatorami zmienności (Parkinsona, Garmana-Klassa i Rogersa--Satchella). W próbie wykorzystano ceny dzienne, przy czym każdy instrument miał 2001 obserwacji i 20-dniowy horyzont prognozy. Dokładność prognoz oceniono za pomocą RMSE i MAE.

Wyniki: Wyniki empiryczne wykazały, że żadne konkretne podejście nie jest powszechnie uważane za lepsze. Zaleca się stosowanie metod naiwnych lub standardowych modeli GARCH, ponieważ są one prostsze niż złożone modele z wybranymi estymatorami i oszczędzają czas obliczeń. Estymatory zmienności poprawiły dokładność w przypadku akcji, ale nie w przypadku innych instrumentów. W przypadku akcji modele oparte na estymatorach uzyskały lepsze wyniki; w przypadku innych instrumentów skuteczniejsze były metody klasyczne.

Implikacje: Praca ta może pomóc badaczom w wyborze odpowiedniego modelu dla konkretnych danych i wskazać, czy zastosowanie innego estymatora wzbogaciłoby wyniki prognoz. Dalsze badania mogłyby oszacować wpływ danych o wyższej częstotliwości na skuteczność estymatorów zmienności.

Oryginalność/wartość: Niniejsze badanie sprawdza, czy polski rynek zareaguje na różne estymatory w sposób spójny z reakcjami obserwowanymi na innych rynkach na całym świecie. Potwierdzono również, że najlepszy model różni się w zależności od instrumentu: model z estymatorem Rogersa--Satchella jest najlepszy dla akcji, GARCH(1,1) dla walut, a metoda historyczna dla towarów.

Słowa kluczowe: zmienność, prognozowanie zmienności, estymatory zmienności, ceny OHLC