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Effectiveness of Monte Carlo Simulations of the S&P 500 Index before and after the Outbreak of the SARS-COV-2 Pandemic*

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Abstract: Risk analysis is an integral part of studying the behavior of financial markets. Crises and emergencies challenge analysts trying to predict the value of stock indices by questioning their assumptions. One such event was the coronavirus pandemic, which undoubtedly affected our economy. The purpose of this study was to examine the impact of the COVID-19 pandemic on the US market and to assess the change in effectiveness of the Monte Carlo method due to the pandemic. The study was realized with 12 MC simulations of daily S&P 500 index prices using historical data from 11.03.2015 to 11.03.2021. The negative impact of the pandemic on the accuracy of MC simulations was observed, lowering the confidence of the results. Changes in sensitivity depending on the chosen time period were also detected. The results may prompt consideration of modifying MC simulations during instability and provide information indicating the use of shorter time series to improve simulation efficiency during crises.

Keywords: Monte Carlo (MC) method, Black-Scholes (B-S) model, index simulation, COVID-19 pandemic.

1. Introduction

Uncertainty and risk on financial markets, resulting from a multitude of macro and microeconomic factors, the behaviour of market participants and random events (Janasz, 2009) create an exceedingly difficult challenge for market analysts. On more than one occasion, the global economy has had to face so-called 'black swans', whose damage often exceeded the threshold of measurable risk in modelled event scenarios (Taleb, 2014). In addition, economic crises often caused a breakdown in the typical and partly predictable behaviour of market participants. As a result, analysts are constantly trying to find better methods of forecasting economic processes that would reduce the risks posed by the possibility of 'black swans' while maintaining the rate of return expected by investors. Today, focusing on automation and the increasing calculating power of computers, Monte Carlo simulations, among others, have become commonplace,

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to which market analysts are able to adapt many financial models. One of the most common is the Black-Scholes model, whose assumptions allow in symbiosis with the above method, to quickly perform thousands of simulations considering various market scenarios. Moreover, the increasing availability of financial data makes it possible to significantly increase the efficiency of such forecasts, thanks to the use of large statistical samples and the central limit theorem (Mitrenga, 2014). However, it is worth noting that Monte Carlo simulations still struggle with the problem of the occurrence of market emergencies, due to their reliance on Gaussian distributions.

An example of an emergency in the financial markets currently dealt with is the SARS-CoV-2 virus pandemic, ongoing since 2020. Moreover, it can be considered an example of a 'black swan'. Therefore, the following was set as a goal: to study the strength of the impact of the coronavirus pandemic on the behaviour of the S&P 500 index and the effectiveness of the Monte Carlo method during the crisis.

2. Impact of the COVID-19 pandemic on financial markets

The coronavirus pandemic declared in 2020 has put the stability of the global financial markets in question. The halting of supply chains, the imposition of restrictions on the number of people in stores and shopping malls and travel restrictions, have set a new standard for the functioning of the global economy and as a result, increased investor uncertainty about the future of many businesses. This uncertainty does not spare the financial markets, whose performance depends closely on the financial capacity of companies and the savings of individuals. A number of studies tried to show the impact of the prevailing pandemic on financial markets in various aspects. Among others, one can point to an analysis of debt instrument markets (Mikita, 2021), which showed an upward trend in total debt issuance in selected EU countries from Q2 2020, which included not only ordinary companies, but also financial market players and local governments, both in developed countries (e.g. France and Spain) and in developing countries (Poland – a 39.7% increase between Q1 2020 and Q4 2020). Another analysed aspectwas the impact of COVID-19 on stock markets (Jaworski and Jaworska-Chodnicka, 2020), which showed a strong and positive reaction of European markets to the announcement of government aid programmes. For US markets, on the other hand, such news meant the reduction in prevailing interest rates. There was also an analysis of the profitability of the banking sector in Poland (Ostrowska, 2021), which revealed a deepening decline in the profitability of Polish banks, particularly evident in 2020, where the ROE ratio fell from 6.84% in the first quarter of 2020 to 3.83% in the last quarter. The above showed the real impact of the pandemic on the behaviour of financial market participants, corporate debt levels and capital market sensitivity. Consequently, this is also an important aspect for analysts trying to forecast the future values of stock market indices, stocks, and corporate financial performance during periods of instability and an indication of which methods retain predictive effectiveness. One should cite here Shaomin Yan and Guang Wu, who investigated the possibility of matching the impact of COVID-19 to the S&P 500 index development by means of random motion using the Black-Scholes model, and showed that the shorter the time window adopted for the simulation, the better the match that could be obtained (Yan and Wu, 2020). Another paper, by Dong-Jin Pyo, used Monte Carlo methods and switching regression to examine the volatility of the Korean stock indices during the coronavirus pandemic, and showed an increase in the effect of asymmetric volatility of the indices (Pyo, 2021). Thus, it can be concluded that the topic of the impact of the pandemic on financial markets from the perspective of stock indices has been addressed in the past, however, it also requires an examination of what is the strength of the impact of the shock caused by the announcement of the pandemic on the effectiveness of Monte Carlo simulations.

3. Research methods – Monte Carlo simulation and Black-Scholes model

In this paper, the Monte Carlo method in conjunction with the Black-Scholes model was used as the research method. Conducting simulations with the above methods required the following steps:

- preparation of the historical closing prices of the index under study and determination of the logarithmic rates of return,
- determination of the standard deviations and drift of calculated rates of return,
- generation of pseudo-random numbers that follow a normal distribution,
- creation of an algorithm that will calculate the simulated closing price.

The determination of logarithmic rates of return can be represented by the following formula:

$$r_l = \ln\left(\frac{K_t}{K_{t-1}}\right),$$

where r_l – the logarithmic rate of return, K_t – the price of the financial instrument in period t, K_{t-1} – the price of the financial instrument in period $t-1$. For the selected time interval of historical data, a set of logarithmic rates of return is determined. The next step is to determine the standard deviation of rates mentioned above from the formula:

$$\sigma = \sqrt{\frac{(x_1 - \bar{X})^2 + (x_2 - \bar{X})^2 + \cdots + (x_n - \bar{X})^2}{n}},$$

where x_n – logarithmic rate of return in the set, $\bar{X}$ – arithmetic mean in the set, n – number of elements in the set. The calculation of the standard deviation is necessary to determine the drift, i.e. the parameter of the expected rate of return on the instrument. The drift is calculated with the following formula:

$$\alpha = \mu + \frac{1}{2}\sigma^2,$$

where μ – the expected value of logarithmic rates of return, σ – the standard deviation of returns. Once a set of returns is created and the standard deviation and drift are calculated, the next step is to execute a loop that generates a pseudo-random number in a normal distribution, calculates the simulated price of the instrument using the formula from the B-S model (Jakubowski, 2011):

$$S_t = S_0 \exp\left(\sigma W_t + \left(\mu + \tfrac{1}{2}\sigma^2\right)t\right),$$

where S_t – stock price at time t, σW_t – stock price volatility at time t, $(\mu + \frac{1}{2}\sigma^2)$ – drift occurring in the process, S_0 – initial price. In this way, a simulated path of the instrument's price movement can be created by adding successive prices to the time series.

Thus, if the generated pseudorandom numbers in a normal distribution are taken as the W_t variable, it will be possible to perform Monte Carlo simulations using the Black-Scholes model (Ziętek-Kwaśniewska, 2006).

Monte Carlo methods were first used in the 1940s to explain complex phenomena occurring in the movement of molecules. The main use of Monte Carlo methods is to describe processes that are too complex to use with other analytical methods (Homa and Mościbrodzka, 2017). Such phenomena are often characterised by the financial markets, where high dynamics and volatility pose a great challenge to analysts, and the multiplicity of factors affecting the formation of the processes describing them makes deterministic modelling much more difficult. The basis for using Monte Carlo methods is to know the probability distribution that characterises the process under study (Bialas, 2012). The idea behind this approach is based on the use of a pseudo-random number generator which draws quasi-random numbers based on the distribution determined (Pawlak, 2012). This allows analysts to estimate the possible development paths of the process under study in the form of the expected value of the simulation performed. The main problem behind this approach, in the case of financial markets, is the frequent lack of certainty about the actual distribution of the studied process and its instability over time, since these markets are extremely susceptible to the influx of information about macro and microeconomic conditions, even the emotions among the market participants (Rudny, 2016). In order to be able to put into practice such a form of forecasting for extremely volatile processes, analysts often use a simplification, which is the assumption about the normality of the distribution of the studied processes (Krawczyk, 2006). This approach can be justified if sufficiently large statistical samples are included. With the increasing availability of data in financial markets, it has become possible to use this method in conjunction with popular financial models, including the Black-Scholes model.

The Black-Scholes model was developed in the 1970s and is used to value derivatives (Black and Scholes, 1973). It is one of the most widely used methods, due to its relative ease of calculation (Chernik, 2013). The B-S model is based on several assumptions, and one of the most important is the assumption that stock price dynamics is based on the Geometric Brownian Motion (Piontek, 2000) in context of applying MC simulation. It is also worth pointing out that the B-S model is not free of limitations and makes several assumptions that are often impossible to meet in practice, such as that the drift and variability of the process under study in the model is constant, which does not coincide with empirical studies and affects the result of the simulation (especially over an extended period). Another limitation is the assumption that companies do not pay dividends, which also affects the valuation of the stock index. Another important assumption rarely reflected in the reality is that market prices move according to Wiener's process. This has a significant impact on the simulation of index prices if the pseudorandom numbers are generated in line with a normal distribution.

Despite the indicated limitations, it is a popular forecasting tool used by stock market analysts in various aspects of financial markets. One can cite here Ziętek-Kwaśniewska, who described the classical application of this method to the valuation of European and American options (Ziętek-Kwaśniewska, 2006). Another application was indicated by Krawczyk, who used the Monte Carlo method to estimate the value at risk for the real estate market (Krawczyk, 2006). These models were also used to forecast stock prices, cf. Estember and Maraña, who assessed the capabilities of Monte Carlo simulation with the Black-Scholes model in valuing shares of Philippine companies (Estember and Maraña, 2016).

4. Analysis of research results

The basis of the Monte Carlo simulations conducted in the paper were the daily closing values of the S&P 500 index for the period: 11.03.2015 – 11.03.2021[1]. Based on the collected data and in accordance with the methodology, sets of logarithmic returns, drifts and expected values of index returns over three consecutive years were determined. In this way, the 3-year, 2-year and 1-year drifts and expected values of returns at the specified four time points were determined:

- one year before the official announcement of the pandemic by the WHO (World Health Organization, 2020),
- on the day the pandemic was announced,
- for one and two years after the WHO announcement.

In this way, it is possible to show the performance of the Monte Carlo simulation both during the 'normal' period of the market development, as well as on the day of the pandemic announcement and after its two-year development. This approach will make it possible to see whether the uncertainty and risk present in the US stock market, deepens as the number of infections increases.

In the next step, after determining the above characteristics, which are the basis of the study to be conducted, simulations of daily closing prices of the S&P 500 index were generated using a customised program developed with the use of statistical libraries[2], commonly employed in financial and statistical analysis in Python[3]. Determining four time points (the day of 11 March in 2019-2022) for which three simulations were performed, each considering historical drifts and expected values in time windows

[1] Platform: Stooq.pl, day: 07.01.2022.

[2] Libraries included: Pandas, Matplotlib, Scipy, Numpy.

[3] The source code can be found in Appendix 1 on page 21.

(from one to three years), 12 simulations were finally conducted. In each of them, 20,000 possible index value paths were generated for 252 trading days from the simulation start date. For each time case considered, the results were illustrated using plots generated in Gretl package:

- a histogram of the frequency distribution of the results of 20,000 simulations per 252 trading days,
- a box plot, including both the mean and median of the results, as well as the actual value of the index.

Based on the obtained graphs and descriptive statistics extracted from the simulations, a comparative analysis of effectiveness of the studied forecasting method was made for the time intervals indicated above. The first simulations analysed were for the forecast as of 11 March 2019, i.e. one year before the pandemic was announced, and the results are presented in Figure 1 and Table 1.

Table 1. Simulation results as of 11.03.2019

Data	11.03.2019		
	11.03.2015- -11.03.2018	11.03.2016- -11.03.2018	11.03.2017- -11.03.2018
Anderson-Darling test (*p*-value)	32.245 (<2.2e-16)	17.611 (<2.2e-16)	22.366 (<2.2e-16)
Expected value of historical rates of return	–0.04%	–0.06%	–0.06%
Historical drift	–0.04%	–0.07%	–0.07%
Simulation starting price (day 11.03.2018)	2 786.57	2 786.57	2 786.57
Actual value (day 11.03.2019; dotted line on histogram and solid line on box plots)	2 783.30	2 783.30	2 783.30
Expected value of the simulation (as of 11.03.2019; dashed line)	2 514.56	2 369.13	2 369.65
μ of simulation + σ (solid line)	2 836.48	2 611.72	2 613.84
μ of simulation – σ (solid line)	2 192.64	2 126.54	2 125.46
Standard deviation of simulation	321.92	242.59	244.19

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

Analysing the results of the MC simulations for the pre-pandemic period (stability in financial markets), it should be noted that for all time windows considered, the distribution of returns has a shape similar to a Gaussian distribution. This could indicate that the index was not characterised by strong asymmetry, as well as 'heavy tails'. However, to be sure whether the distribution is normal, goodness-of-fit test should be performed. The Anderson-Darling test was chosen as this approach is particularly sensitive to heavy tails and skewness (Nelson, 1998). The results from normality tests indicate that no distributions of simulation results are normal as the p-values for all of them are close to 0. It can be observed that even before the start of the pandemic, distribution of S&P 500 index was not normal. Therefore, the MC simulation method based on the B-S model could not be applied effectively. For the sake of this study, it was assumed that distribution before COVID-19 was normal in order to investigate what is the level of impact of the pandemic on the standardised Monte Carlo approach with normal distribution in place. The simulation based on the 3-year expected value and drift has the smallest forecasting error – only in this case the actual value was within +/- one standard deviation of the expected value of the forecast. For the other simulations, the index price was above the upper limit of the range. It is worth noting that for the two-year and one-year data ranges, the standard deviations were lower.

This means that 2015 was characterised by much higher volatility than the other years in the included time window, since only the simulation with a three-year drift included historical data from this period. The reason for this may have been the deficient performance of the energy and fossil fuel sector – the value of the S&P 500 Energy index fell by more than 23% at the time (Standard & Poor's, 2022). This decline was caused by extraordinary declines in oil prices (USA Today, 2016). However, because the time windows studied were still close to normal distribution, analysts were able to correctly apply the Monte Carlo simulation in the context of studying S&P 500 index.

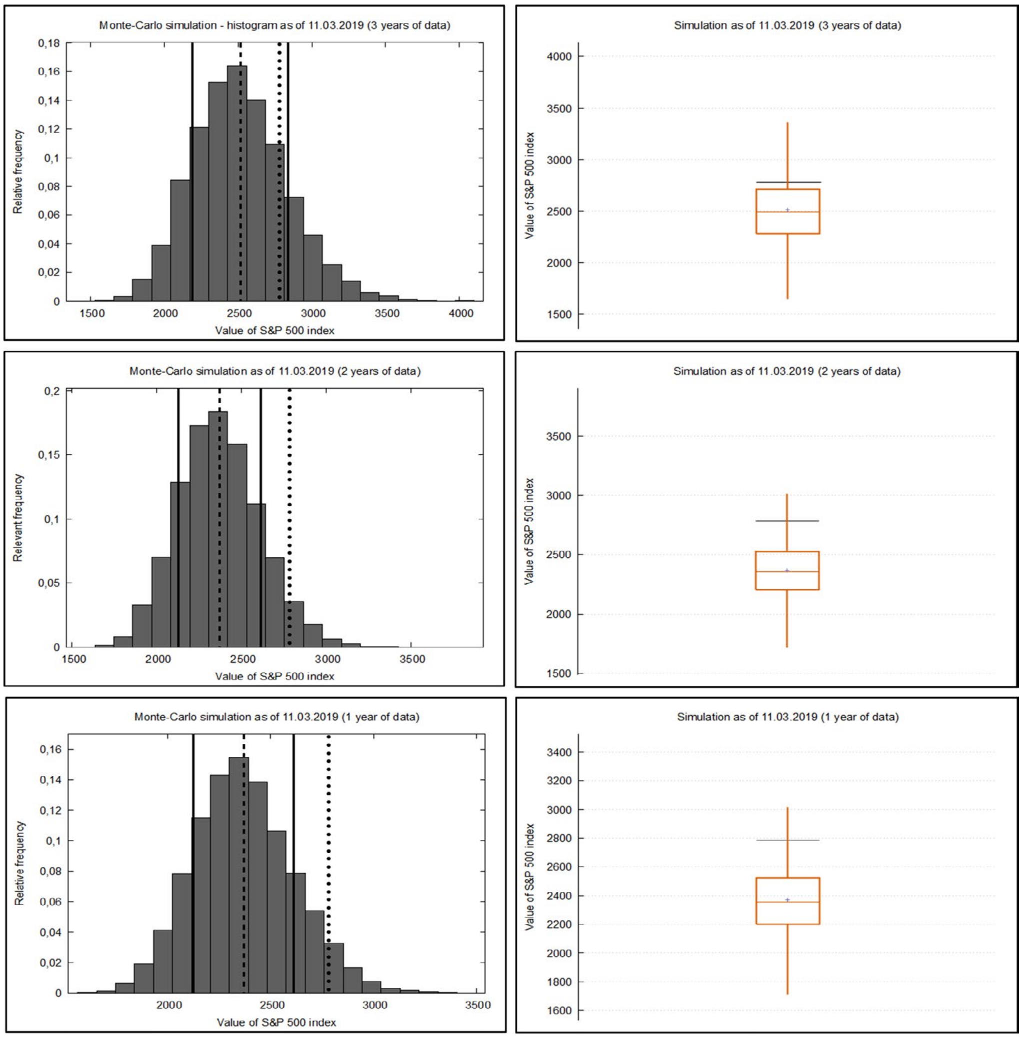

Fig. 1. Simulations of the value of the S&P 500 index as of 11.03.2019 in three time windows

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

The next simulations involve forecasting the value of the index as of 11 March 2020, the date the WHO announced the pandemic, and the results are shown in Figure 2 and Table 2.

Table 2. Simulation results as of 11.03.2020

Data	11.03.2020		
	11.03.2016- -11.03.2019	11.03.2017- -11.03.2019	11.03.2018- -11.03.2019
Anderson-Darling test (*p*-value)	28,315 (<2,2e-16)	44,564 (<2.2e-16)	49,437 (<2,2e-16)
Expected value of historical rates of return	–0.042%	–0.032%	–0.00004%
Historical drift	–0.046%	–0.035%	–0.00526%
Simulation starting price (day 11.03.2019)	2 783.30	2 783.30	2 783.30
Actual value (day 11.03.2020; dotted line on histogram and black line on box plot)	2 741.38	2741.38	2741.38
Expected value of the simulation (as of 11.03.2020; dashed line)	2 498.42	2 570.83	2 784.14
μ of simulation + σ (solid line)	2 810.19	2 921.26	3 237.01
μ of simulation – σ (solid line)	2 186.65	2 220.40	2 331.27
Standard deviation of simulation	311.77	350.43	452.87

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

The histograms are still nearly symmetrical and show the visual characteristics of a normal distribution, just as in the case of the simulation for 2019. However, the A-D tests for all the investigated periods show that all the distributions of the simulations are different from Gaussian. Therefore, the MC method still could not be applied effectively for this period. The smallest error was observed in the simulation with a one-year drift, for which the expected value was remarkably close to the actual value of the index as of 11 March 2020. This is quite different from the forecasts for 2019. The expected values of the other two simulations are within 50% of all generated observations, which was also not the case in the previous forecasts. Another difference between the 2019 and 2020 forecasts is the value of the standard deviations. It can be pointed out that for the 2020 forecasts, the uncertainty of the simulation results increases as the range of historical data shortens towards the date of the pandemic's announcement.

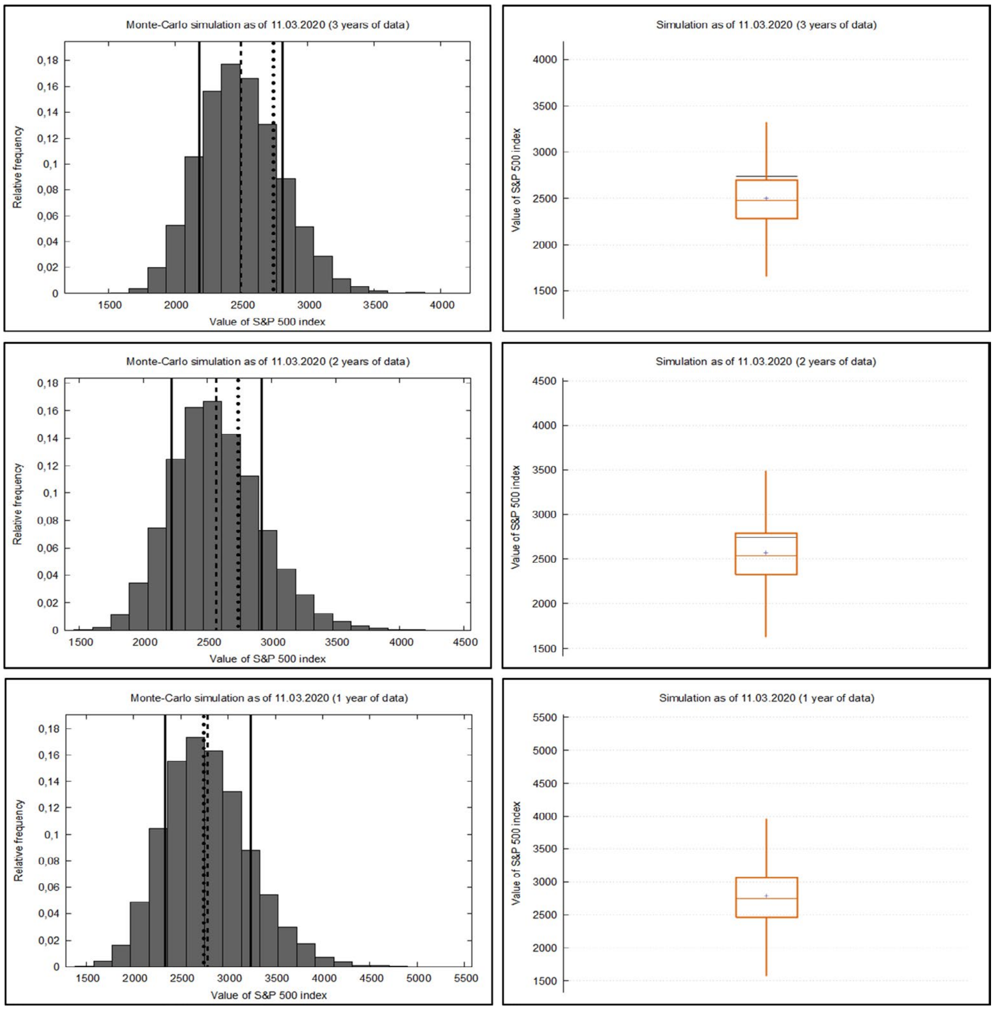

Fig. 2. Simulations of the value of the S&P 500 index as of 11.03.2020 in three time windows

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

This is an interesting phenomenon, since the shock from the increase in infections has not yet occurred in the input data. The high standard deviation for the one-year time window is also interesting, as reports of the first cases of infection in China did not appear until November 2019, and the simulation only includes data up to March of the same year. A plausible reason for the increased volatility could have been growing investor uncertainty due to the announcements of the US presidential candidates and the ongoing trade war between the US and China (Warsaw Institute, 2021). Thus, there is no basis for the possibility that the COVID pandemic may have affected the results of the 2020 forecasts. Therefore, to show possible changes in the effectiveness of the Monte Carlo method due to the outbreak of the pandemic, simulations would also need to be performed for further periods that take into account the time of the market shock.

The next simulations involve forecasting the value of the S&P 500 index as of 11 March 2021, and the results are shown in Figure 3 and Table 3.

Table 3. Simulation results as of 11.03.2021

Data	11.03.2021		
	11.03.2017- -11.03.2020	11.03.2018- -11.03.2020	11.03.2019- -11.03.2020
Anderson-Darling test (p-value)	55.574 (<2.2e-16)	55.347 (<2.2e-16)	67.785 (<2.2e-16)
Expected value of historical rates of return	–0.019%	0.003%	0.006%
Historical drift	–0.024%	–0.003%	–0.001%
Simulation starting price (day 11.03.2020)	2 741.38	2 741.38	2 741.38
Actual value (day 11.03.2021; dotted line on histogram and black line on box plot)	3 939.34	3 939.34	3 939.34
Expected value of the simulation (as of 11.03.2021; dashed line)	2 608.84	2 760.28	2 781.18
μ of simulation + σ (solid line)	3 016.66	3 245.99	3 307.19
μ of simulation – σ (solid line)	2 201.02	2 274.57	2 255.17
Standard deviation of simulation	407.82	485.71	526.01

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

Analysing the graphs for the 2021 forecasts, one can point out the elongated right tails of the histograms, which means the occurrence of extremely high and rare forecast index values. Therefore, one can already see the preliminary impact of the crisis on the frequency distribution of simulation results after including Q1 2020 in the historical data. Moreover, the box plots are significantly underestimated relative to the actual value of the index, in contrast to previous simulations. This may indicate a decrease in the efficiency of the Monte Carlo method for the period under study. The observed situation is markedly different from the simulations for 2019 and 2020. The Anderson-Darling test confirms the lack of normality of the results of the distributions of the simulations for 2021. After extracting descriptive statistics from the performed forecasts, a significant change in the standard deviations can be observed in relation to the simulations for previous years. The largest difference was characterised by the simulation with a two-year drift and standard deviation – about 135 points, which gives an increase of 38.6% in the standard deviation relative to 2020. The shock is evident in terms of the distribution of the data itself, as well as the volatility of the process studied. The reaction of investors was very sharp, in the context of the studied crisis, as the announcement of the pandemic became an inflammatory factor for preventive and relief measures by governments around the world (including the United States), which had a significant impact on the functioning of the global economy and rising concerns about the future of operations in all industries.

These observations demonstrate that the shock of the pandemic announcement had a significant and negative impact on the effectiveness and predictive ability of the MC method. The final issue worth exploring is to evaluate whether the pandemic has caused a shock that maintains increased uncertainty in subsequent years.

The latest simulations are projections as of 11 March 2022. Figure 4 and Table 4 show how the index projections behave after a year from the announced start of the pandemic.

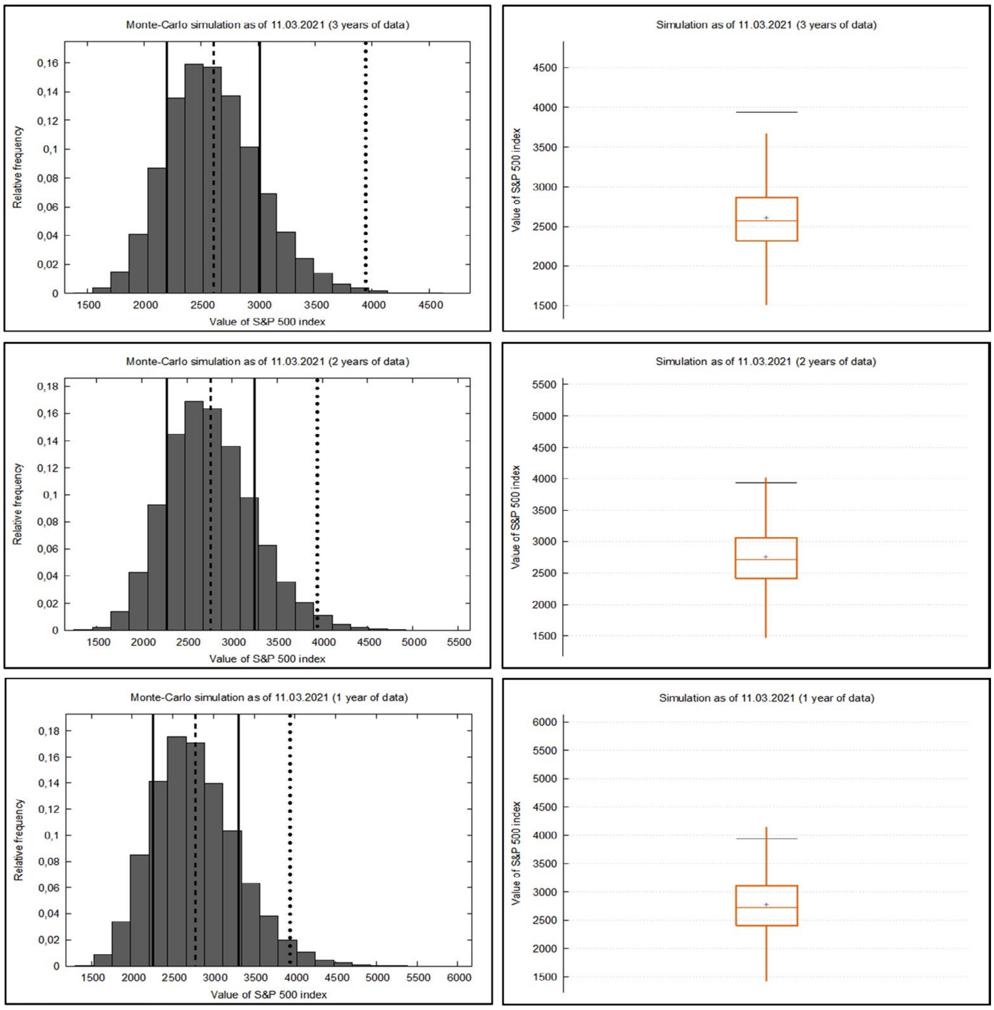

Fig. 3. Simulations of the value of the S&P 500 index as of 11.03.2021 in three time windows

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

Table 4. Simulation results as of 11.03.2022

Data	11.03.2022		
	11.03.2018- -11.03.2021	11.03.2016- -11.03.2018	11.03.2020- -11.03.2021
Anderson-Darling test (p-value)	106.41 (<2.2e-16)	134.06 (<2.2e-16)	202.76 (<2.2e-16)
Expected value of historical rates of return	–0.05%	–0.07%	–0.14%
Historical drift	–0.06%	–0.08%	–0.16%
Simulation starting price (day 11.03.2021)	3 939.34	3 939.34	3 939.34
Actual value (day 11.03.2022)	–	–	–
Expected value of the simulation (as of 11.03.2022; dashed line)	3 504.63	3 312.66	2 736.81
μ of simulation + σ (solid line)	4 339.02	4 196.95	3 629.24
μ of simulation – σ (solid line)	2 670.24	2 428.37	1 844.38
Standard deviation of simulation	834.39	884.29	892.43

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

All the histograms indicate that the distributions of the simulation results for 2022 are asymmetric and left-skewed, so they deviate significantly from the characteristics of a Gaussian distribution. In contrast to the simulation results for 2019 and 2020, the deformation of S&P 500 distribution is fully visible. In addition, the phenomenon of leptokurtosis, which justifies the higher value of the standard deviation, is becoming increasingly evident. Thus, it can be concluded that there was a collapse in the market, and the Monte Carlo method lost its predictive effectiveness. In 2021 in the US market, the historical returns of the S&P 500 index had to deviate significantly from typical values, and their probability distribution began to change drastically. Market information also confirms the deepening crisis caused by the coronavirus pandemic. The World Health Organization indicated that between 2020 and 2021, there were 14.9 million deaths due to the COVID-19 pandemic globally, 84% of which occurred in Southeast Asia, Europe and North and South America (World Health Organization, 2022). In addition, global supply chains operated by major US companies such as Microsoft and Apple were disrupted by the imposed restrictions in China (Forbes, 2020), lowering expectations for the operational efficiency of these companies. The uniqueness of the pandemic crisis is also evidenced by the reactions of the rating agencies. In the period 01.2020-07.2020, S&P Global downgraded 1015 entities in North America alone, and the total number of rating downgrades globally was similar to the number of downgrades in 2009, following the outbreak of the global financial crisis in 2007 (Reuters, 2020). The projections conducted reflect the negative effects of the ongoing pandemic and show the extent to which market information distorts the typical distribution of returns in the US market.

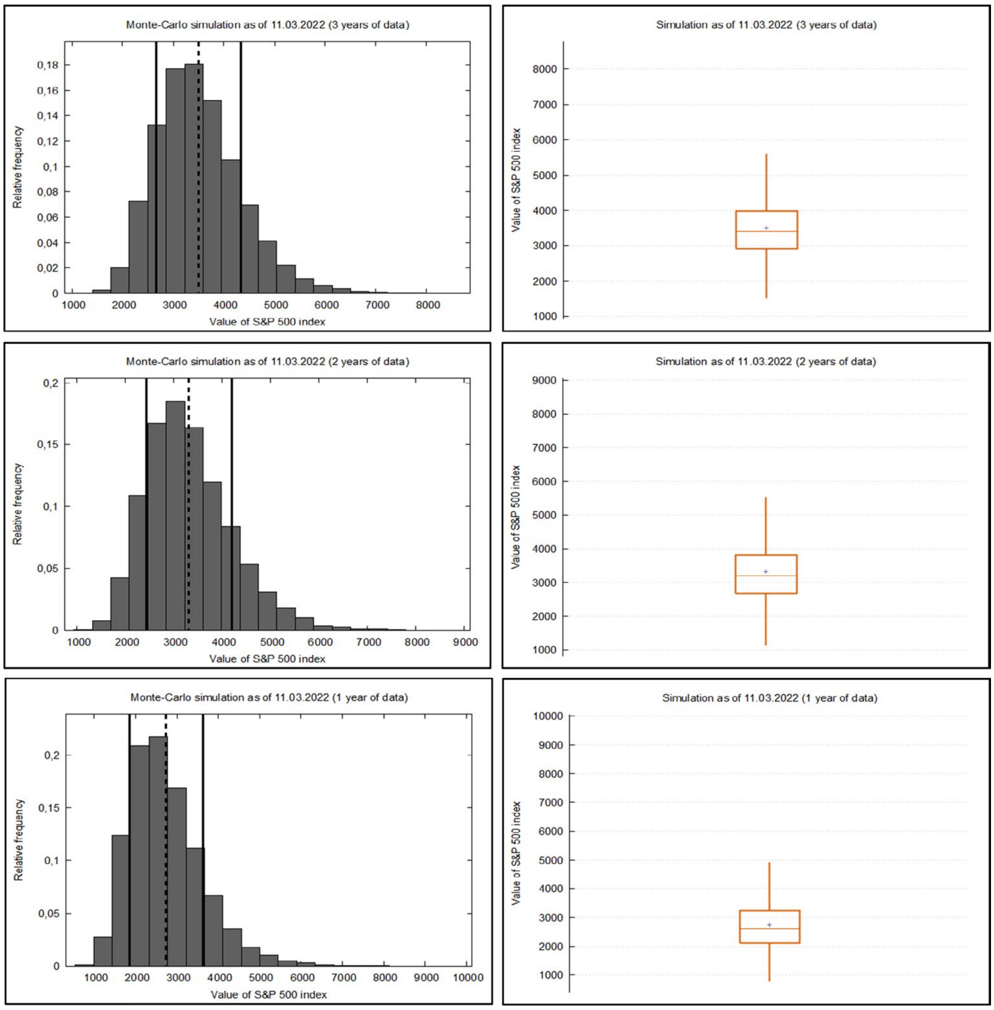

Fig. 4. Simulations of the value of the S&P 500 index as of 11.03.2022 in three time windows

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

Moreover, the left-skewed, leptokurtic histograms and significantly higher standard deviations demonstrate that not only did the shock maintain higher uncertainty in the market, but the uncertainty further increased as the crisis deepened.

With all the periods studied, the efficiency of the MC method based on the Black-Scholes model dynamically decreased. Figure 5 shows a summary of the results of all 12 simulations.

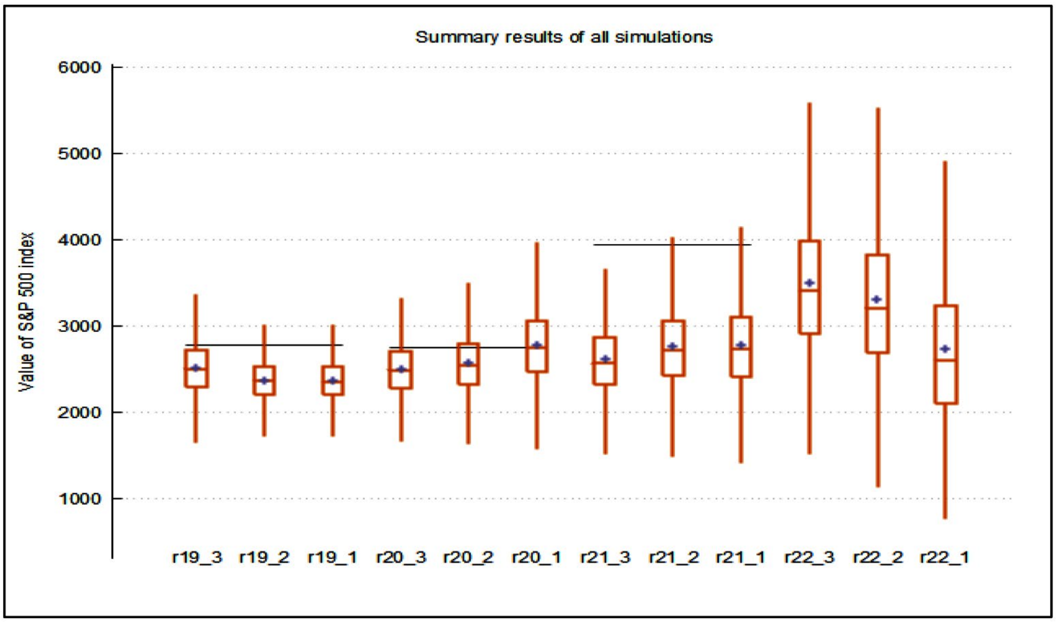

Fig. 5. Summary of Monte Carlo simulation results (black lines = actual values of the index)

Source: own study based on data from www.stooq.pl platform (access: 01.11.2021).

Analysing the box plots, the range of simulation values obtained increases with time. For the simulation with a 3-year drift for 2019, the range between the third and first quartiles was 428.97 points, and for the corresponding simulation for 2022, the value was already 1072.8. This result shows that uncertainty in the market deepened and the accuracy of the simulation results decreased, as the range of possible results increased by about 150%. This also shows that the effectiveness of the Monte Carlo method decreased in the event of an emergency and may be a signal for market analysts to revise models based on this method. Another phenomenon observed is the difference in the sensitivity of capturing changes in the range of results of the generated simulations. As the range of historical data widens, the simulations are less sensitive to the changes taking place in the market, and show more shocks when capturing a pandemic crisis. For the 3-year data range, this insensitivity can be seen in the Q3-Q1 intervals, which almost doubled in the simulation for the last year relative to 2021. On the other hand, for the 2-year data range, one can already see the greater sensitivity of changes in these intervals, where the change was about 77% between 2021 and 2022. For the shortest data range, the changes were even smaller, presenting change in intervals at about 61.6%. The reason for the reduced sensitivity of simulations with a wider range of data may be the effect of older results on the drift and expected value. When observing the impact of the COVID-19 pandemic on uncertainty in the US market, it seems that the best option is to adopt simulations for the shortest data range, as it is most sensitive to changes caused by macro and microeconomic information. Therefore, it can be concluded that the announcement of the coronavirus pandemic may have affected the efficiency of MC simulations, causing the range of possible outcomes to increase by about 61.6% if the shortest data range is applied.

5. Conclusion

The analysis of the generated Monte Carlo simulations in terms of the formation of probability distributions and descriptive statistics of the range of results obtained, showed a significant and negative change in the confidence with which the value of the S&P 500 index can be estimated. The standard deviations of the simulations studied showed an increasing trend as the COVID-19 pandemic progressed, which also increased the range of possible estimation results, and thus the possible failure to meet the one-sigma rule. In addition, the difference between the actual value and the expected value of the simulation for 2021 was shown to be significantly larger than for the previous periods. Given that the starting point was the day the pandemic was announced, this may indicate that investors in the US market, through their actions, have already begun to show their uncertainty and risk aversion towards the market by observing the global increase in the number of coronavirus infections. Furthermore, for the simulations of the preceding years, the accuracy of the results relative to the actual values was shown to be significantly higher, confirming that MC simulations perform well for estimations during normal market periods. In the case of the juxtaposition of all the simulations conducted, the analysis of the box plots showed the induction of a shock in the context of the possible range of S&P 500 values for 2022. The change in the Q3-Q1 interval between 2021 and 2022 was in the estimated range of 61.6%-100% (depending on historical data range) which indicates a definite decrease in the efficiency of forecasting using MC methods. The final observation is also the occurrence of the difference in the sensitivity of the simulations performed, depending on the adopted data range. The analysis showed that the shorter the range of data adopted, the more sensitive to substantial changes the Monte Carlo simulation of the S&P 500 index. This may be a hint for US market analysts to use shorter ranges of data for forecasting using MC simulations in periods of crisis or extraordinary macroeconomic events, to better show the actual reaction of the market and to estimate the tested values with greater awareness.

The above conclusions show that the effectiveness of MC simulations of the S&P 500 index significantly decreased after the announcement of the coronavirus pandemic. This situation may prompt further research and testing of various modifications to the Monte Carlo simulation to see if they will positively affect the efficiency of the estimation when using this method for existing crises. Modifications may include using a different probability distribution, such as the stable Levy distribution, which, according to some studies (Duczkowski, 2021), shows better efficiency in explaining market processes, particularly financial markets.

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Appendix 1

Source code for generating Monte Carlo simulations

```
# importing statistical libraries needed for MC analysis (requires installing first)
import numpy as np
import pandas as pd
from pandas_datareader import data as wb
import matplotlib.pyplot as plt
from scipy.stats import norm
# functions from the pandas library to record historical index prices from the Stooq platform; the start and end variables are adjusted depending on the range of data covered in the paper
df = wb.DataReader('^SPX', data_source='stooq', start='2020-3-11', end='2021-3-11')
# calculation of logarithmic rates of return, average rate of return and standard deviation and drift
logarithmic_returns = np.log(1 + df['Close'].pct_change())
mu_r = logarithmic_returns.mean()
std_r = logarithmic_returns.std()
drift = mu_r - 0.5*(std_r**2)
wyniki = []
# function that generates 20,000 rates of return consecutive trading days according to Monte Carlo simulation
for i in range(20000):
	d_returns = np.exp(drift + std_r * norm.ppf(np.random.rand(252)))
	list_prices = [df['Close'][0]]
# function that calculates the new price after taking into account the generated rate of return from the simulation
	for x in d_returns:
		list_prices.append(list_prices[-1]*x)
	plt.plot(list_prices)
	wyniki.append(list_prices)
	lista=[]
# A loop that adds the results to the time series to be visualized
	for z in results:
		lista.append(z[-1])
# Data visualization functions from matplotlib library - price paths, title changed according to selected time range
plt.axhline(df['Close'][0],color='r', linestyle='solid',linewidth=2)
plt.xlabel(„Dni")
plt.ylabel(„Wartość indeksu SP500")
plt.title(„Symulacja ruchu wartości indeksu SP500 na dzień 2022-3-11 (1 rok danych)")
plt.show()
# functions that calculate the mean and standard deviation of simulation results
m_pri = np.mean(lista)
dev_one = np.std(lista)
dev_two = dev_one * 2
# functions that generate a histogram with the frequency distribution of the results, bins, that is the number of intervals selected to show the corresponding ranges of the values of the results obtained
plt.hist(lista,bins=20)
plt.axvline(m_pri + dev_one,color='b',linestyle='dashed',linewidth=2)
plt.axvline(m_pri - dev_one,color='b',linestyle='dashed',linewidth=2)
plt.axvline(np.mean(lista),color='g',linestyle='dashed',linewidth=2)
```

```
plt.axvline(df['Close'][0],color='r',linestyle='dashed',linewidth=2)
plt.xlabel(„Wartość indeksu SP500")
plt.ylabel(„Częstość")
plt.title(„Symulacja Monte-Carlo - histogram dla SP500")
plt.show()
# function which generates box plot of simulation's results
plt.boxplot(lista, showmeans=True)
# function that generates simulation results in text form, date changed according to the simulation day
being analyzed
print(„Prognoza na 11-3-2022")
print(„Mu_r: „, str(mu_r))
print("Drift: ", str(drift))
print("Actual price (red): ", str(df['Close'][0]))
print("Expected price (green): ", str(m_pri))
print("Std dev +: ", str(m_pri + dev_one))
print("Std dev -: ", str(m_pri - dev_one))
plt.show()
# function which saves simulation results to .csv file
df = pd.DataFrame(lista)
df.to_csv('2022_1.csv')
```

Efektywność symulacji Monte Carlo indeksu S&P 500 przed ogłoszeniem i po ogłoszeniu pandemii SARS-COV-2

Streszczenie: Analiza ryzyka jest integralną częścią badania zachowania rynków finansowych. Sytuacje kryzysowe stanowią wyzwanie dla analityków próbujących przewidzieć wartość indeksów giełdowych, kwestionując ich założenia. Jednym z takich wydarzeń była pandemia koronawirusa, która niewątpliwie wpłynęła na naszą gospodarkę. Celem niniejszego badania było zbadanie wpływu pandemii COVID-19 na rynek USA oraz ocena zmiany efektywności metody Monte Carlo na skutek pandemii. Badanie zostało zrealizowane za pomocą 12 symulacji MC dziennych cen indeksu S&P 500 przy użyciu danych historycznych między 11.03.2015 a 11.03.2021. Zaobserwowano negatywny wpływ pandemii na efektywność symulacji, obniżający wiarygodność wyników. Wykryto również zmiany czułości w zależności od wybranego okresu. Wyniki mogą skłonić do rozważenia modyfikacji symulacji MC podczas braku stabilności i dostarczyć informacji skłaniających do skorzystania z krótszych szeregów czasowych w celu poprawy efektywności symulacji podczas kryzysów.

Słowa kluczowe: metoda Monte Carlo, model Blacka-Scholesa, symulacja indeksu, pandemia COVID-19.