

Should One Assume the Discount Rate to Be One of the Risk Factors?

Jan Kaczmarzyk

University of Economics in Katowice

e-mail: jan.kaczmarzyk@ue.katowice.pl

ORCID: 0000-0002-4139-8858

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Abstract: The Monte Carlo simulation is the ultimate solution for considering nearly all possible scenarios in presumably any discounted cash flow valuation. This paper argues that a discount rate expresses an investor's current requirement and should be respectively perceived as a parameter only. The consequences of qualifying a required rate of return (a discount rate) as a risk factor in a discounted cash flow valuation are described in the paper using a free cash flow financial model of an asset being a hypothetical publicly traded enterprise. The case study is a discounted cash flow valuation using the Monte Carlo simulation for risk analysis. The various sets of assumptions are considered to explain the consequences of qualifying a required rate of return in a discounted cash flow model as a risk factor. As indicated in the paper, the discount rate as an additional risk factor with an attributed probability distribution increases the volatility of a risk variable, then the distribution of a risk variable becomes more flattened. In previous studies, some authors indicated that a discount rate could be considered a risk factor in the Monte Carlo simulation (Krysiak 2000; Damodaran 2018).

Keywords: corporate finance, valuation, DCF, risk analysis, Monte Carlo simulation

1. Introduction

Financial models implemented in corporate finance mostly rely on the discounted cash flow valuation approach. The basic concepts behind various forms of a discounted cash flow valuation are the present value of an asset and the net present value of an asset. The author used the present value concept to estimate the value of an asset (e.g. in an enterprise valuation, a bond or a credit valuation, a derivative valuation, etc.) whereas we specifically focus on a net present value to assess the profitability of an asset (e.g. in an investment project profitability appraisal). Both concepts use a discount rate as a required rate of return for investors or broadly investors and lenders together depending on a particular analytical situation and the type of free cash flow stream.

The traditional approach to discounted cash flow valuation results in a single scenario built on the analyst's specific assumptions. However, a volatile economic environment suggests intuitively an

infinite number of scenarios that might happen in the real world. The ultimate solution for considering nearly all possible scenarios in presumably any discounted cash flow valuation is the Monte Carlo simulation (Brealey et al., 2014; Golden and Golden, 1987; Hertz, 1964; Kroese et al., 2014). In terms of risk analysis, risk factors are those financial model inputs that are subject to the economic environment and its volatility. The others are only parameters. Nonetheless, there is an infinite number of risk factor scenarios, while there is no more than one scenario of an input being a parameter.

Some studies indicated that a discount rate could be considered a risk factor (Damodaran, 2018; Krysiak, 2000), whereas this author argues that a discount rate expresses an investor's current requirement and should be respectively perceived as a parameter. The paper shows the consequences of taking up an assumption in the Monte Carlo simulation which would qualify a discount rate as a risk factor.

2. Theoretical Background

Although the difference between present value and net present value is technically very simple, it is conceptually truly significant. The present value concept is based on a cash flow stream that excludes a possible payment or a series of possible payments for an asset. Estimating the present value of an asset gives an analyst a possible value of an asset – a hypothetical payment. If an investor paid the present value for an asset and received the exact estimated cash flow stream in the considered time horizon, they would certainly achieve their required rate of return being here the internal rate of return of the transaction. One can conclude that the present value is a value of an asset that resembles both its ability to generate a certain cash flow stream and the investor's profitability requirement. Net present value additionally considers a payment or a series of payments for an asset (e.g. a market price of a stock or a bond, a capital expenditure or expenditures related to an investment project, etc.) and depicts the profitability of a transaction. The basic rule here is widely known and accepted – if a payment for an asset was lower (higher) than its present value, the net present value would be positive (negative) and the investor would realize an internal rate of return higher (lower) than their required rate of return. The concepts of present value, net present value, and internal rate of return are quite dated (e.g. Fischer, 1930; Gordon, 1955; Lorie and Savage, 1955; Williams, 1938) but they remain truly relevant for modern finance. The present value concept has successfully been the core of the value-based management idea so far (Rappaport, 1999; Rappaport, 2006), whereas the net present value concept is deservedly classified as one of the most important concepts in modern finance (Brealey et al., 2014).

An analyst who decides to perform a discounted cash flow valuation must estimate 1) a cash flow stream and 2) a required rate of return. The question is whether cash flow stream components and a required rate of return are all risk factors. Beginning with the definition of a risk factor, a risk factor is technically a financial model input variable that may vary from its forecast value due to the economic environment and its volatility. Assuming that a required rate of return was a risk factor would mean that an investor would be uncertain about the level of the satisfying required rate of return. An investor is actually always uncertain about the final payment or the series of payments – not uncertain about their own required rate of return at the moment of valuation. Therefore, a discounted cash flow valuation is performed to find the present value which could assure the investor's required rate of return; thus the investor may decide whether the offered payment is acceptable.

In terms of own equity, one thinks about shareholders' required rates of return. These rates were successfully estimated using the dividend model (Gordon and Shapiro, 1956) or the CAPM (Lintner, 1965; Mossin, 1966; Sharpe, 1964; Treynor 1962) for assets being stocks of publicly traded enterprises so far. For non-publicly traded companies, usually the CAPM with Hamada's correction was involved (Damodaran, 2012; Hamada, 1972). The dividend model derives a required rate of return as the internal

rate of return of a cash flow stream including the current price of a stock and an infinite series of dividends. Through the CAPM a required rate of return is the sum of a risk-free rate and the difference between a market risk rate and the risk-free rate adjusted by systemic risk. Hamada's equation changes the required rate of return to reflect the financial leverage of a particular enterprise. A required rate of return derived this way is mostly affected by the historical period considered or the interval chosen. Therefore, it may differ depending on subjective assumptions of a particular financial analyst and may be even, easily manipulated – but it still helps to find a reasonable and widely accepted level of a required rate of return in a particular analytical situation. The fact that a required rate of return derived through the mentioned or similar models may differ due to the period and/or the interval of the historical data taken into account, does not mean that a required rate of return of an investor is somehow volatile at the moment of valuation. It is just an estimated investor's requirement, broadly – a minimum value of a required rate of return they could have accepted.

A cash flow stream is simultaneously affected by numerous risk factors and parameters. Therefore, as there is no single scenario of a cash flow stream that may happen in the future – there is logically an infinite number of possibilities due to infinite combinations of possible values of risk factors – due to infinite future states, the economic environment may fall into. The solution for considering nearly all possible scenarios of cash flows in a discounted cash flow valuation is the Monte Carlo simulation (Hertz, 1964; Kroese et al., 2014). In fact, this simulation is not quite new and was not even designed specifically for finance (Ulam et al., 1947) but is very relevant and useful. The scenarios of cash flows are the direct source of the present value and net present value scenarios respectively. As a consequence, the outcome of the Monte Carlo simulation is then the probability distribution of possible present values or net present values (or the other cash flow-based profitability measures).

The Monte Carlo simulation results in a probability distribution of a risk variable. Typically, risk factors are financial or non-financial figures like unit prices, unit costs, exchange rates, interest rates, fixed costs, unit demands, etc. Whenever the Monte Carlo simulation is involved, every input variable being a risk factor must be depicted with a probability distribution chosen objectively, quasi-objectively, or subjectively. Due to interdependencies between them, a correlation matrix must be specified too (Kaczmarzyk, 2016; compare Hull, 2018; Vose, 2008). The result of the Monte Carlo simulation is the probability distribution of a risk variable which reflects simultaneous, interdependent, and non-linear changes in risk factors. The concept of qualifying the discount rate as an additional risk factor (Damodaran, 2018; Krysiak, 2000,) results in attributing an additional probability distribution. This could be a uniform probability distribution (e.g. Damodaran, 2018, p. 35) or a triangle probability distribution (e.g. Krysiak, 2000, p. 68). The variability of a discount rate will affect the volatility of a risk variable and blur the influence of the other risk factors. The aim of the paper was to show how a required rate of return qualified as a risk factor affects the result of a DCF analysis.

3. Methodology

The consequences of qualifying a required rate of return (a discount rate) as a risk factor in a discounted cash flow valuation are shown in the paper using a free-cash-flow-to-firm financial model of an asset being a hypothetical publicly traded enterprise. The model estimates the intrinsic value per share and is based directly on the present value concept (Figure 1).

The model starts with the revenue forecast (Row 4) with a certain level of revenue growth rate (Row 3). The operational costs less depreciation (Row 5), fixed assets (Row 10), and net working capital (Row 13) are derived from the respective ratios indicating their relation to the revenues (Rows 5, 7, and 12). Then, the cash flow stream (Row 23) is calculated. The required rate of return in the model is the weighted average cost of capital (Row 28). The cash flow stream is affected by two undisputable

risk factors: the revenue growth rate and the ratio of operational costs less depreciation as well as by the questionable risk factor – the required rate of return. The revenue growth rate and the ratio of operational costs less depreciation are assumed to be the same in every single period of the detailed projection. The model output is the intrinsic value of a single stock.

	A	B	C	D	E	F	G	H
			t0	t1	t2	t3	t4	t5
1								
2								
3	Risk factor	Revenue growth rate		5.00%	5.00%	5.00%	5.00%	5.00%
4		Revenues	150,000.00	157,500.00	165,375.00	173,643.75	182,325.94	191,442.23
5	Risk factor	Operational cost ratio		73.00%	73.00%	73.00%	73.00%	73.00%
6		Operational costs less depreciation	73,000.00	114,975.00	120,723.75	126,759.94	133,097.93	139,752.83
7		Net fixed assets to revenues		27.00%	27.00%	27.00%	27.00%	27.00%
8		Net fixed assets	27,000.00	42,525.00	44,651.25	46,883.81	49,228.00	51,689.40
9		Depreciation rate		9.00%	9.00%	9.00%	9.00%	9.00%
10		Fixed assets	29,670.33	46,730.77	49,067.31	51,520.67	54,096.71	56,801.54
11		Depreciation	2,670.33	4,205.77	4,416.06	4,636.86	4,868.70	5,112.14
12		Net working capital to revenues		0.07	0.07	0.07	0.07	0.07
13		Net working capital	7,000.00	11,025.00	11,576.25	12,155.06	12,762.82	13,400.96
14								
15		EBIT		38,319.23	40,235.19	42,246.95	44,359.30	46,577.26
16		Tax rate		19.00%	19.00%	19.00%	19.00%	19.00%
17		Tax		7,280.65	7,644.69	8,026.92	8,428.27	8,849.68
18		NOPAT		31,038.58	32,590.51	34,220.03	35,931.03	37,727.58
19		Depreciation		4,205.77	4,416.06	4,636.86	4,868.70	5,112.14
20		Fixed assets change		- 17,060.44	- 2,336.54	- 2,453.37	- 2,576.03	- 2,704.84
21		Working capital change		- 4,025.00	- 551.25	- 578.81	- 607.75	- 638.14
22			0	1	2	3	4	5
23		Cash flow		14,158.91	34,118.78	35,824.71	37,615.95	39,496.75
24		Terminal value						394,967.47
25		Discounted cash flow		12,871.73	28,197.33	26,915.64	25,692.20	269,768.10
26								
27		Cash flow growth rate for t>5		0.00%				
28	Risk factor	Required rate of return		10.00%				
29								
30		Enterprise value	363,445.00					
31		Financial liabilities present value	260,000.00					
32		Net enterprise value	103,445.00					
33								
34		Shares	1,000.00					
35	Risk variable	Intrinsic value		103.45				
36		Market value		90.00				
37								

Fig. 1. The financial model

Source: own elaboration.

Table 1. Sets of assumptions

Set 1	Fixed required rate	Fixed revenue growth rate	Fixed operational cost ratio
Set 2	Uniform required rate	Fixed revenue growth rate	Fixed operational cost ratio
Set 3	Triangular required rate	Fixed revenue growth rate	Fixed operational cost ratio
Set 4	Normal required rate	Fixed revenue growth rate	Fixed operational cost ratio
Set 5	Fixed required rate	Uniform revenue growth rate	Fixed operational cost ratio
Set 6	Fixed required rate	Triangular revenue growth rate	Fixed operational cost ratio
Set 7	Fixed required rate	Normal revenue growth rate	Fixed operational cost ratio
Set 8	Fixed required rate	Fixed revenue growth rate	Uniform operational cost ratio
Set 9	Fixed required rate	Fixed revenue growth rate	Triangular operational cost ratio
Set 10	Fixed required rate	Fixed revenue growth rate	Normal operational cost ratio
Set 11	Fixed required rate	Normal revenue growth rate	Normal operational cost ratio
Set 12	Uniform required rate	Normal revenue growth rate	Normal operational cost ratio
Set 13	Fixed required rate (lower)	Normal revenue growth rate	Normal operational cost ratio
Set 14	Fixed required rate (higher)	Normal revenue growth rate	Normal operational cost ratio

Source: own elaboration.

The case study is a discounted cash flow valuation using the Monte Carlo simulation for risk analysis. The various sets of assumptions (Table 1) are considered to explain the consequences of qualifying a required rate of return in a discounted cash flow model as a risk factor. The model was developed using Microsoft Excel 365 and the simulations performed using Palisade @RISK 8.2.1.

4. Case Study

The first set of assumptions (Figure 2) qualifies only the required rate of return as a risk factor to depict how a fixed required rate of return may affect the valuation. Obviously, the fixed required rate of return level results in the fixed intrinsic value level (Set 1). The higher (lower) the required rate of return level, the lower (higher) the intrinsic value of an enterprise would occur.

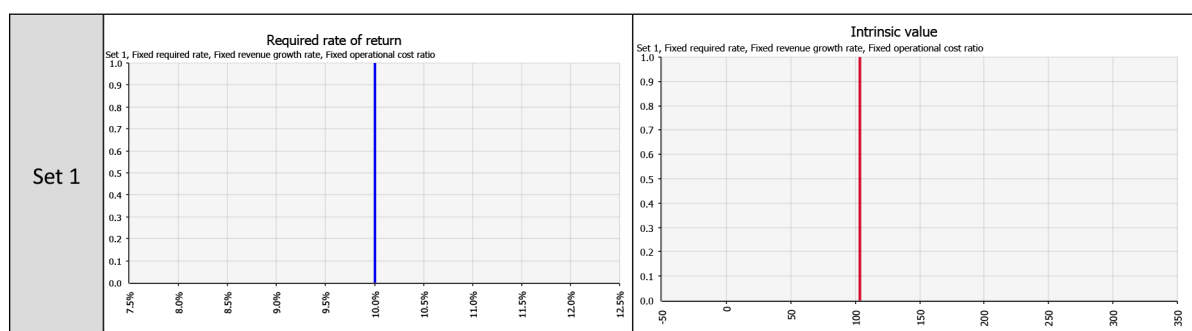


Fig. 2. Fixed required rate of return, fixed revenue growth rate, and fixed operational cost ratio

Source: own elaboration.

If the required rate of return of the investor was assumed to be a risk factor, it would reflect a very hypothetical situation in which the investor would be uncertain about an adequate required rate of return level (Figure 3). The investor could assume then, e.g. that their required rate of return belonged to a certain range but every level within this range had the same probability. To reflect such a phenomenon, a uniform probability for the required rate of return should have been chosen (Set 2). The probability distribution of the intrinsic value precisely reflects the way how a certain level of the required rate of return affects the intrinsic value. Namely, the required rate of return affects the intrinsic value in a nonlinear way. For the same relative increase or decrease of the initial required rate of return level there occurs an incommensurate change in the intrinsic value. Moreover, the probability of the intrinsic value shows precisely that the lowest intrinsic value has the highest probability of occurrence, whereas the highest intrinsic value has the lowest probability. The phenomenon of the nonlinear type of interdependency between the required rate of the return level and the respective intrinsic value would also be clearly visible if the investor applied another type of theoretical probability distribution to reflect their beliefs. For example, they could use a simple triangular probability distribution to reflect their belief that an adequate required rate of return belonged to a certain range in which extreme values had the lowest probability and there was a certain level of the required rate of return between them with the highest chance of occurrence from the investor's point of view (Set 3).

The way in which a required rate of return affects an intrinsic value would be a little bit less visible but still relevant if a normal probability distribution was chosen (Set 4) to reflect investor's requirements. It should be emphasised that a symmetrical required rate of return probability distribution generally results in an asymmetrical probability distribution of the intrinsic value.

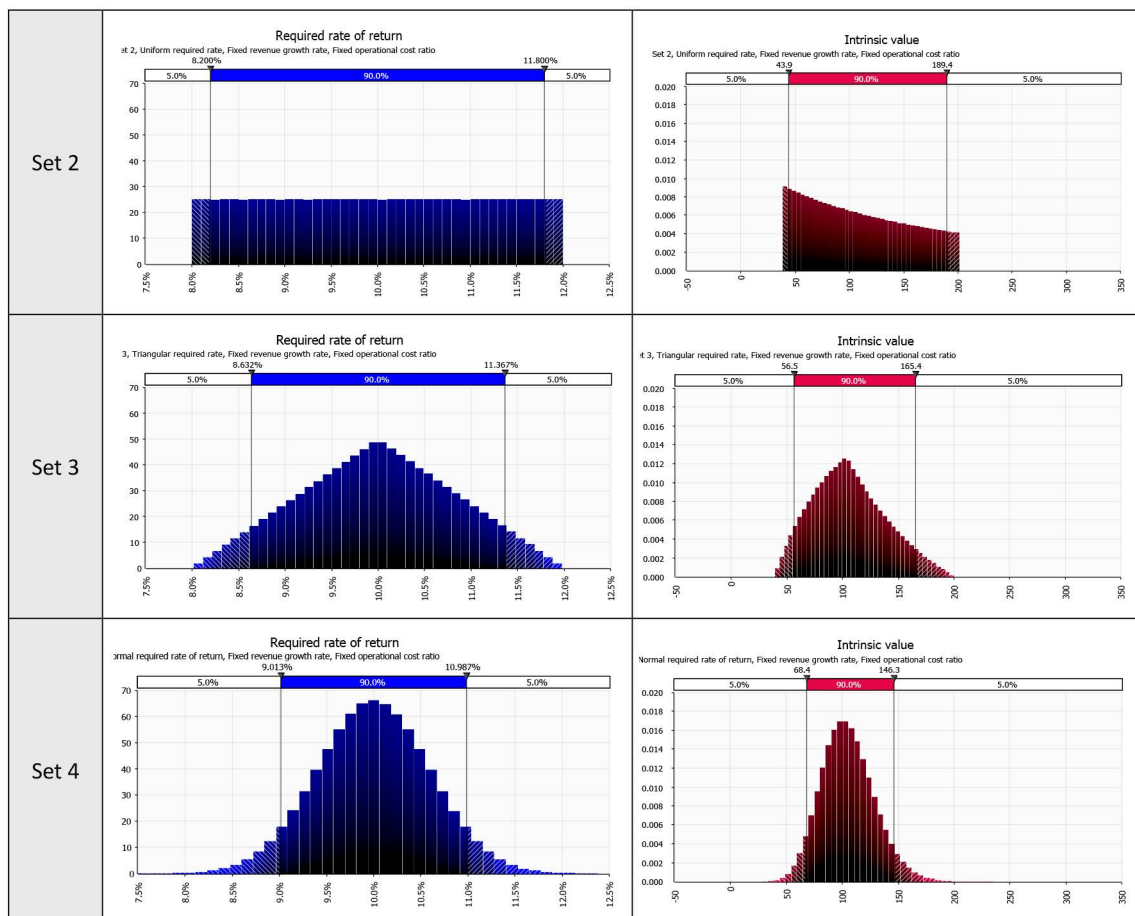


Fig. 3. Variable required rate of return, fixed revenue growth rate, and fixed operational cost ratio

Source: own elaboration.

The interpretation of the intrinsic value probability distribution would change significantly when the investor had a fixed required rate of return but assumed that the growth of revenues could be the only risk factor (Figure 4). This would be a reasonable assumption because, at the moment of valuation, the investor would not know how the economic environment could develop and how this all might affect the entrepreneurial activity of their enterprise. An infinite number of possible revenue growth paths results in the respective infinite number of intrinsic value scenarios. Different probability distributions represent different types of possible investor's assumptions (Sets 5, 6 and 7). The impact of the growth rate level on the intrinsic value is also nonlinear which is worth emphasising here.

Assuming that the ratio of the operational costs was a risk factor would also be reasonable. The probability distribution of the intrinsic value is symmetrical due to possible changes in the ratio of the operational costs (Figure 5).

The Monte Carlo simulation is used in finance especially to examine the influence of the simultaneous impact of risk factors. The volatility of the intrinsic value would rise if both risk factors were taken into account at the same time (Figure 6, Set 11). If the required rate of return was added as a third risk factor (e.g. uniformly distributed), the volatility of the intrinsic value would grow again (Figure 6, Set 12). Although technically possible, such an approach leads to a blurred picture of risk. The volatility of the economic environment is raised by uncertain expectations of an investor.

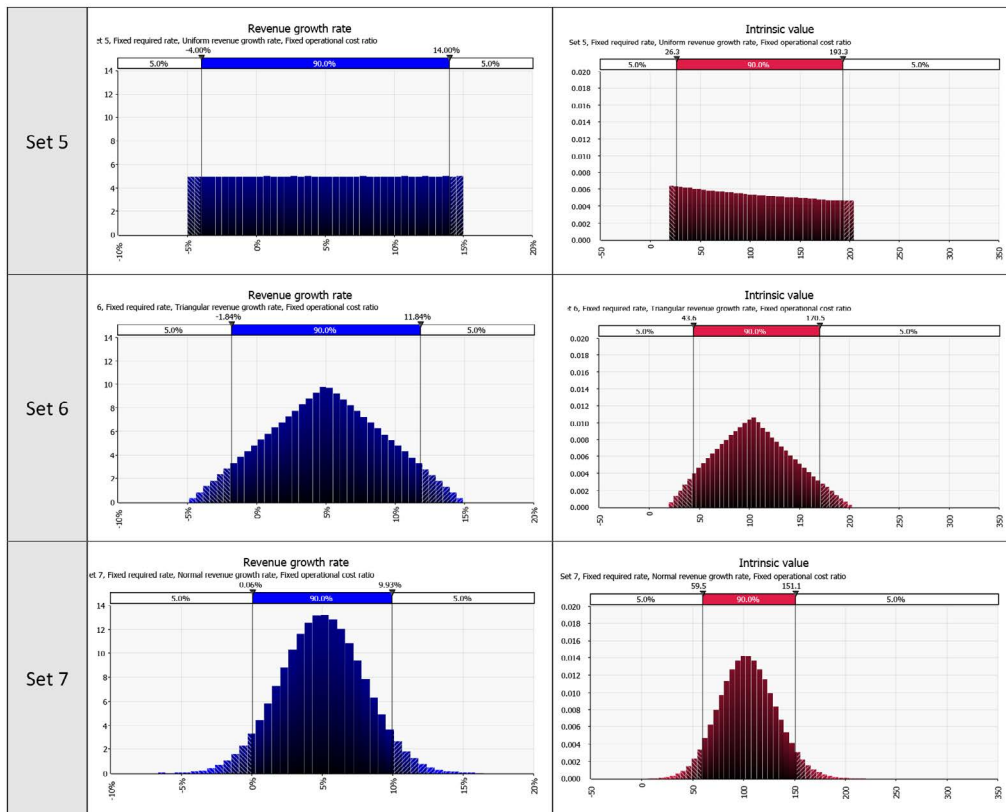


Fig. 4. Fixed required rate of return, variable revenue growth rate, and fixed operational costs ratio

Source: own elaboration.

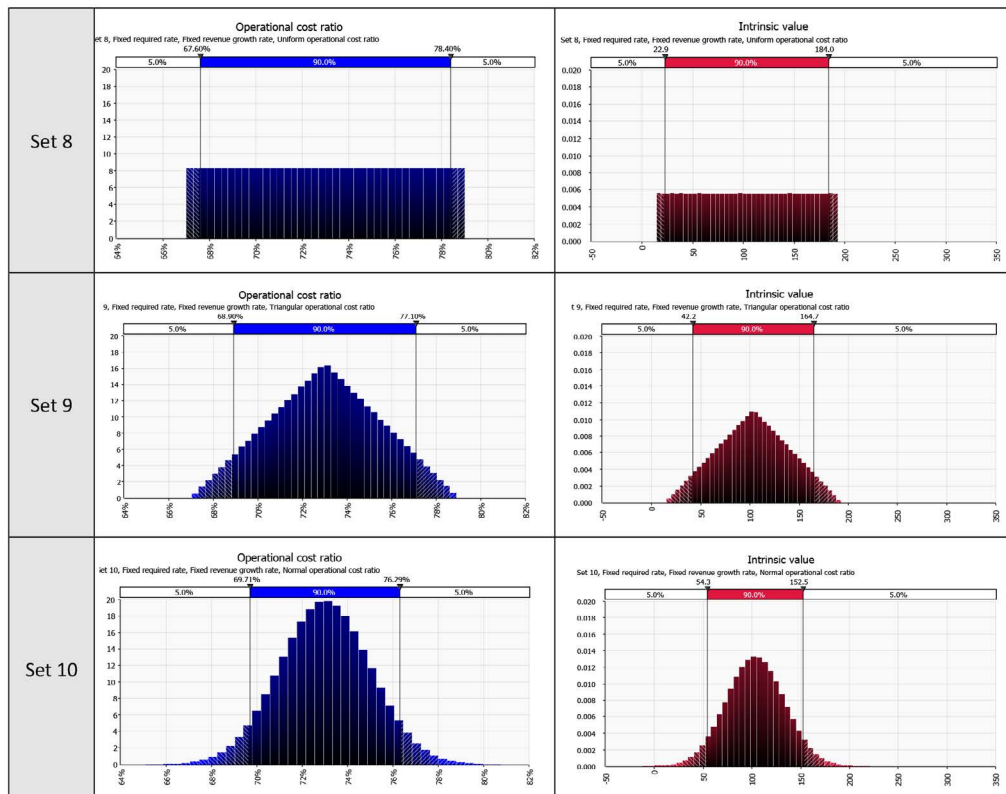


Fig. 5. Fixed required rate of return, fixed revenue growth rate, and variable operational cost ratio

Source: own elaboration.

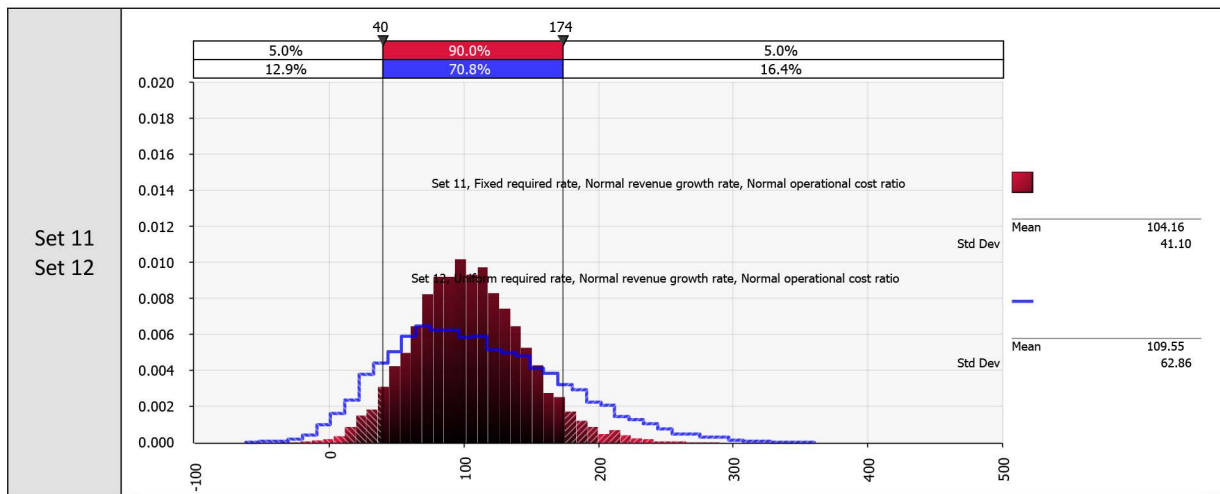


Fig. 6. Fixed and variable required rate of return, variable revenue growth rate, and variable operational cost ratio

Source: own elaboration.

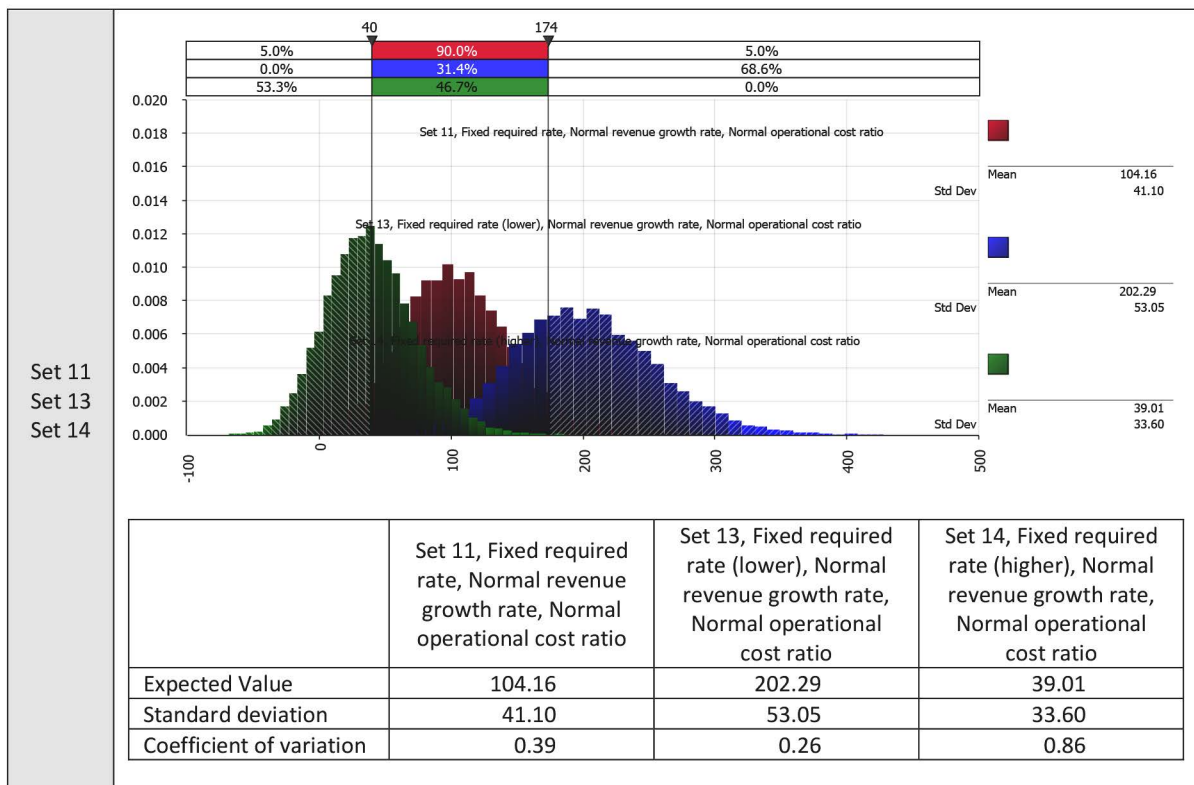


Fig. 7. Different fixed required rates of return, variable revenue growth rate and variable operational cost ratio

Source: own elaboration.

Even though a volatile required rate of return blurs the risk associated with an asset, it somehow is a volatile input because it may differ among different investors valuing an asset with different required rates of return. Having set the same assumptions for all risk factors besides the required

rate of return (Figure 7), the investor with the lowest required rate of return will face the highest absolute (lowest relative) volatility of the intrinsic value (Set 13), whereas the investor with the highest – will face the lowest (highest) volatility (Set 14).

5. Conclusion

The discount rate as a risk factor with an attributed probability distribution affects a risk variable in a nonlinear way. This can be easily confirmed by qualifying the discount rate as the only risk factor during simulation. Regarding a uniformly distributed discount rate, namely a discount rate with a certain range of possible values with the same probability of occurrence, one can expect different probabilities of a risk variable, such as an intrinsic value. Depending on a risk factor, influence can be nonlinear (e.g. the growth rate of revenues) or linear (e.g. the share of variable costs). The discount rate as an additional risk factor with an attributed probability distribution increases the volatility of a risk variable. Then, the distribution of a risk variable becomes more flattened.

In the case of different discount rate levels representing different investors with different expectations, one must emphasise that the higher the discount rate, the lower the present value, and consequently, the lower the intrinsic value. At the same time, the lower the absolute volatility, the higher the relative volatility of the present value, and consequently, the intrinsic value.

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Czy powinniśmy zakładać, że stopa dyskontowa jest jednym z czynników ryzyka?

Streszczenie: Symulacja Monte Carlo jest narzędziem umożliwiającym rozważenie „prawie wszystkich” scenariuszy w dowolnej wycenie/ocenie wykorzystującej zdyskontowane przepływy pieniężne i uwzględniającej ryzyko. Zdaniem autora stopa dyskontowa jest wyrazem bieżących żądań w zakresie rentowności. Należy ją zatem postrzegać jako parametr. Konsekwencje kwalifikacji wymaganej stopy zwrotu (dyskontowej) jako czynnika ryzyka w wycenie zobrazowano w artykule z wykorzystaniem modelu finansowego aktywa stanowiącego hipotetyczne przedsiębiorstwo notowane, wykorzystującego wolne przepływy pieniężne. Studium przypadku stanowi wycenę aktywa z uwzględnieniem ryzyka poprzez implementację symulacji Monte Carlo. W celu zobrazowania kwalifikacji stopy dyskontowej jako czynnika ryzyka rozważono różne zestawy założeń dla symulacji. Należy stwierdzić, że stopa dyskontowa stanowiąca dodatkowy czynnik ryzyka z przypisanym rozkładem prawdopodobieństwa zwiększa zmienność zmiennej ryzyka. Rozkład zmiennej ryzyka ulega spłaszczeniu. W dotychczasowych badaniach wskazywano, że stopa dyskontowa może być traktowana jako czynnik ryzyka w analizie ryzyka z wykorzystaniem symulacji Monte Carlo (Damodaran, 2018; Krysiak, 2000).

Słowa kluczowe: finanse przedsiębiorstwa, wycena, DCF, analiza ryzyka, Monte Carlo
