ECONOMETRICS
EKONOMETRIA
Advances in Applied Data Analysis

Econometrics. Ekonometria.
Advances in Applied Data Analysis
Year 2026, Vol. 30, No. 1
ISSN 2449-9994
journals.ue.wroc.pl/eada

Partitions and Surroundings

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Quote as: Biernacki, M., & Smoluk, A. (2026). Partitions and Surroundings*. Econometrics. Ekonometria. Advances in Applied Data Analysis*, *30*(1), 41-46.

DOI: 10.15611/eada.2026.1.03

JEL: C02

Abstract

Aim: This paper gives a strict definition of a partition, and a recursive formula for the number of partitions is provided. The infinite matrix of conditional partitions plays an essential role in the proof.

Methodology: Special attention is paid to the connection of partitions with discriminations. The preferences and the partitions determined by them lead to a conjecture generalising the 2/3 rule of Łyka.

Results: Toffler waves are extended to decreasing geometric sequences of circular arcs. Time is round and can be measured by an angle. Eternity is identical to the current moment.

Originality/value: This is an attempt to provide a mathematical explanation for Toffler waves.

Keywords: partition, Chinese partitions, discrimination, preference, permutation, cycle, time

1. Introduction

We live in a world of partitions. Partitions are everywhere. A partition is a dish on a plate, cars standing in a parking lot, books on shelves, book chapters, words, etc. One cannot live without mathematics. A bill from a store is a partition of a sum, and a family is a partition of several people into individual age classes, genders, and degrees of kinship. Two apples and three pears on a plate are also a partition. Mathematics is a science about the physical world, not a pure abstraction. The content of the article is connected with two valuable studies which partly reflect the spirit of this presentation (Ross & Wright, 2000; Graham et al., 2012). To illustrate what formal definitions can be, the study defined a sequence of natural numbers (a_n) , where

$$a_n = 1 + (n!)^{n!},\ n \in N.$$

What is the number a_{1000}? An object defined formally has no cognitive value. It may be a prime number or a composite number, yet in our world and in the entire cosmos of the model such a number has no realisation.

2. Definition and Theorem

What is the formal definition of a partition? A partition is a finite, weakly increasing sequence of nonzero natural numbers. A finite sequence of numbers $a = (a_0, a_1, .., a_k)$, where a_i belongs to *N*, and *N* denotes the semigroup of natural numbers, is a partition of the natural number $n = a_0 + a_1 + a_2 + \cdots + a_k$, if a_0 is nonzero and $a_i \leq a_{i+1}$. The number zero has no partition.

Let *Pn* denote the set of partitions of natural number *n*, p_n is the number of partitions of *n*. It follows that $p_0 = 0, p_1 = 1$; the number one has only one partition (1), the number two has two partitions $P_2 = \{(1,1), (2)\}$, hence $p_2 = 2$. The set of partitions of the number five is

$P_5 = \{(1,1,1,1,1), (1,1,1,2), (1,1,3), (1,2,2), (1,4), (2, 3), (5)\}$, i.e. $p_5 = 7$.

Quantity p_{ij} denotes the cardinal number of the set of conditional partitions, i.e. the set $\{p \in P_i\colon i_0 \geq j$, $i_0 = \min(Im(p))\}$, where

$$Im(p) = \{i_0, \dots, i_k\}, i = i_0 + \cdots + i_k.$$

Theorem. The recursive formula gives the number of partitions of the number *n*+1:

$$If\ n + 1 = 2i + 1\ or\ n + 1 = 2i, then\ p_{n+1} = 1 + \sum_{k=0}^{i} p_{n-k,k+1}.$$

Proof. The number of partitions p_{n+1} is one partition (n+1) plus all partitions of number n with a leading 1, plus those partitions of number n-1 for which the minimum of the image is at least 2 with a leading 2, plus those partitions of number n-2 for which the minimum is at least 3 with a leading 3, etc. The proof is complete.

To better illustrate the idea of the proof, calculate again the number of partitions of the number 5. The partitions of 5 are: one partition (5), five partitions of the number 4: (1,1,1,1), (1,1,2), (1,3), (2,2), (4), and one partition of the number 3 – (3). From these partitions, one obtains 7 partitions of the number 5. In the recursive formula for the number of partitions, matrix P of size p_{ij} plays the essential role. This is an infinite, triangular matrix: one writes it – omitting the first column and the first row consisting of all zeros – differently than is traditional. The main diagonal of this matrix is filled with ones (1s), while the elements of the first column p_{n1} are the numbers of partitions n: $p_n = p_{n1}$.

$$P = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 22 & 7 & 3 & 2 & 1 & 1 & 1 & 1 & \cdots \\ 15 & 4 & 2 & 1 & 1 & 1 & 1 & 0 & \cdots \\ 11 & 4 & 2 & 1 & 1 & 1 & 1 & 0 & \cdots \\ 7 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & \cdots \\ 5 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \end{pmatrix}$$

Partitions are a good monetary system, weights, precision scales, and probabilistic models. The study's cognition is connected with partitions. One does not see all the numbers: one sees zero – an empty set, one sees one, one sees two, and one can see three more. One does not see the number four, hence it is calculated using partitions (1,3) and (2, 2). This is the case with larger numbers: seven is (2,2), i.e. a partition of the numbers 4 and 3, so number 7 is a partition (2, 2, 3).

3. Discriminations and Partitions

Partitions are related to discriminations of sets. A discriminant of a set corresponds to a single partition, but a partition corresponds to many equivalent discriminants. A permutation of a finite set is a one-to-one function transforming the set into itself. A permutation is a product of disjoint cyclic permutations; cyclic permutations determine the discriminant of the permuted set, thus a permutation determines a partition.

Similarly, a preference in a finite set. Preference – a reflexive and transitive relation – generates an equivalence relation: a is equivalent to b if and only if a prefers b and b prefers a. The abstraction classes of this relation create discriminants in the domain of preferences. A preference also determines its partition.

One can say that a preference satisfies the Condorcet condition if the most numerous subset of the discrimination determined by it contains less than 1/3 of the elements of the set – the domain of this preference. If a set of preferences in a finite set of elements is given, then the maximal relation created by this set of preferences is called the relation that contains the most frequently appearing pairs in these preferences; both pairs ab and ba may belong to the maximal relation (Łyko & Smoluk, 2014).

Hypothesis: If linear preferences satisfy the Condorcet condition and the maximal relation satisfies the Łyko condition, then the maximal relation is a linear preference.

This is a generalisation of Łyko's 2/3 rule (Łyko, 2000) relating to linear orders. The linearity of the order and preferences means that any two elements are comparable: ab or ba is an element of the relation. The Łyko condition implies that the frequency of maximal elements is strongly greater than 2/3 (Maciuk & Smoluk, 2018).

4. Chinese Partitions and Probability

The three-point decomposition (5/22, 7/22, 10/22) is a partition (5, 7, 10) of 22. This decomposition is an approximation of the ideal decomposition, which is a concept of pure mathematics defined by the triple $\left(\frac{1}{1+\sqrt{2}+2}, \frac{\sqrt{2}}{1+\sqrt{2}+2}, \frac{2}{1+\sqrt{2}+2}\right)$, when √2 is replaced by its approximation 7/5. Every discrete, finite distribution can be reduced to an urn model: in this case, it will be an urn containing 5 white, 7 red, and 10 black balls. The ratio of the weight of the yolk in the egg to the weight of the white and the yolk is probably the golden number chi. A hen's egg determines the partition (2, 3) of the number 5: 2 parts of the egg are yolk and 3 are white. The golden number relates the proportions in Greek temples to the egg eaten for breakfast.

Special partitions related to the Chinese remainder theorem are called Chinese partitions. The Chinese partition of n is the partition (r, s, …, s), i.e. $n = k \cdot s + r$. Using the remainder theorem, Chinese generals calculated a unit's strength not by counting them off but by lining them up and looking at the remainder. This was probably due to the soldiers' clumsiness and keeping the size of the unit a secret.

For the reader's convenience, the authors recall this beautiful theorem. Let Z_n denote the ring of residues modulo n in the ring of integers Z. If the numbers n_1, .., n_k are pairwise coprime, then the system of equations $x = a_i$, where a_i belongs to Z_{ni}, $i = 1,\dots, k$ is consistent.

5. Toffler Waves and Partitions

Partitions are also Toffler waves. Toffler divided civilisation into periods that decrease as humanity progresses. Natural development, according to Malthus' theory, is exponential. The famous equation says that the growth of the process is proportional to the current state

$$y' = \rho \cdot y.$$

Exponential development cannot last long for obvious reasons: crises will appear – resources will be limited – and there will be a regression. This is the physical meaning of Toffler's spiritual theory – emotional crises. In his view, the third wave is a period of collapse. Information on the division of historical time proposed by Toffler allows to conclude that the periods form a geometric sequence with a quotient of less than one, this is probably the golden ratio $\chi = \frac{\sqrt{5}-1}{2}$ or its measurable approximation 2/3. Following Toffler's suggestion, the authors assumed the year 1000 as the beginning of the current development. This year can be considered the end of antiquity and the creation of modern states. The dependence of secular power on ecclesiastical power also decreases. The next characteristic point of human development is the year of great geographical discoveries. One conventionally assume that it was the year 1450. The year 1750 was the time of intensive activity of Parisian literary salons and the formation of modern society was close to Toffler's date – the year 1800 – steam engines and industry. It follows that the first term of the mentioned geometric sequence a_0 = 1450 – 1000 = 450. The second term a_1 = 1750 – 1450 = 300. It follows that the quotient of this geometric sequence is 2/3. Therefore one obtains a relationship

$$a_{n+1} = \frac{2}{3} \cdot a_n, n \in N.$$

The recursive formula calculates the next transition points between different periods of social development. In Toffler's case, the third period ended with a shock. It is commonly believed that the fifth wave was decisive. By definition, a_2 = 2/3 300 = 200, which gives the date 1950 = 1750 + 200. This year marked the development of the atomic era and computers. Indeed, there was a shock, but not a devastating one. The size of a_3 = 2/3 200 = 133, hence around 2083 humankind can expect great changes. Further arcs become shorter and shorter because it is a decreasing geometric sequence. Flat time is circular, so time is an angle, and its measure is a radian. The geometric sequence defined above divides the circle's circumference into decreasing arcs – analogs of Toffler waves. In practice, a_n infinite sequence is always finite. A finite number of expressions of a geometric sequence is a partition. The partitions depend on the accuracy of the calculation, as shown in the probability distribution given above. The sequence (a_n) must be normalised by dividing each word by the circumference of a circle of an appropriately chosen radius. Partitions 'sting' history. The beginning of time, t_0 can be chosen arbitrarily, and the moment marking the n-th period is denoted by t_n. The number a_n denotes the arc of the circle, and the number t_n is the time at which the corresponding period ends. The angle determined by this arc is time (Figure 1). The geometric sequence adds up to the circle's circumference, while the times (t_n) form a convergent sequence. This sequence is strongly increasing: $t_0 = 0$, $t_{n+1} = a_n + t_n$, and at the same time there is a paradoxical equality

$$t_0 = \lim_{n \to \infty} t_n.$$

A sequence growing on the circle of a circle has the property that it first moves away from the origin and then moves closer to it. The order of the circle is, in fact, a preference. The initial moment t_0 coincides with the limit, which can be considered eternity. Here, one can recall a famous anecdote related to time and eternity. A salon savant asked Einstein: *Professor*, *what is the difference between time and eternity*? – *Simple: I need time to explain, and you need eternity to understand.* Circular time is eternity, and linear time is physical time. The difference between eternity and time is like the difference between a circle of a circle and a straight line. Both of these topological spaces are locally homeomorphic, but they are not globally isomorphic. The circle is a compact space, while the straight line is not a complete space – the ends are missing. A one-point compactification of the straight line is, of course, a circle.

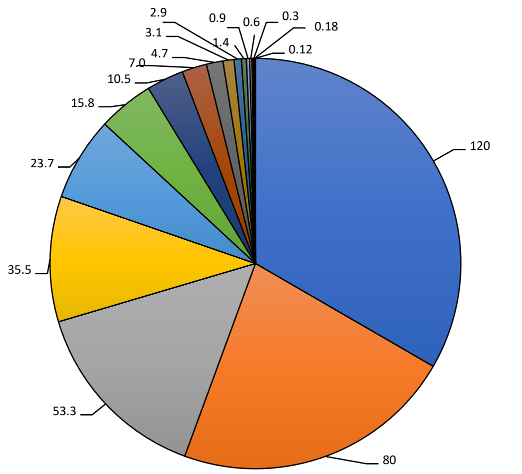

Fig. 1. Growth and crisis. Malthus and Toffler

Source: own study.

6. Conclusion

We live in a post-industrial society. The whole world is a global village. People are comparing each other to each other. The New York fable is told everywhere. The same is true of all tastes. We live not in a material world but in a virtual *simulacrum*. We use the world loosely – being *homo ludens*. We have become slaves to technology. We constantly crave novelty, and never like Tantalus – this desire will not be satisfied. We have golden prosperity, but still – like Midas – we are hungry.

We see the progress of science in our daily lives. We live better, eat better, know the world's remotest corners, and explore space and the depths of the oceans. Almost every day we receive new information, which sometimes pleases us and sometimes stresses us. *The impact of* the news is overwhelming. In Toffler's terms, this is a shock – a *future shock*. History provides examples of the destruction of machines and factories, caused by increased productivity and loss of wages for manual labour. Structural changes in industry also required the transformation of the workforce into new areas. What will be the end of our unbridled development, heralded by the 1939 World Exposition in New York? Its slogan was *Building the World of Tomorrow*. President Roosevelt and Albert Einstein opened that exhibition. Will this development end in a war with robots – computer dwarfs, or will we live in paradise, with sky-high *gnomes* serving us?

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Partycje i otoczenie

Streszczenie

Cel: W artykule podano ścisłą definicję partycji oraz dowiedziono rekurencyjnego wzoru na liczbę partycji. Istotną rolę w dowodzie odgrywa nieskończona macierz partycji warunkowych.

Metodyka: Zwraca się szczególną uwagę na związek partycji z dyskryminantami. Preferencje i wyznaczone przez nie partycje prowadzą do hipotezy uogólniającej zasadę 2/3 J. Łyki.

Wyniki: Fale Tofflera rozszerzono na malejące ciągi geometryczne łuków okręgu. Czas jest okrągły i może być mierzony kątem. Wieczność jest tożsama z chwilą bieżącą.

Oryginalność/wartość: Jest to próba uzasadnienia matematycznego fal Tofflera.

Słowa kluczowe: partycja, partycje chińskie, dyskryminacja, preferencja, permutacja, cykl, czas