Default Prediction Using the Cox Regression Model and Macroeconomic Conditions – A Lifetime Perspective

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DOI: 10.15611/eada.2024.2.04
JEL: C1, C11

Abstract

Aim: Since the implementation of International Financial Reporting Standards 9 (IFRS 9), several techniques on estimating the risk parameters for calculating the expected credit losses (ECL) have been implemented across financial institutions. The purpose of this study was to present the advantages of using survival analysis for the estimation of the probability of default (PD) given the particularity of the method, within the estimation of the time up to an event occurring.

Methodology: The Cox Proportional Hazard Rate was selected as the model to predict the default incorporating the time to event and the macroeconomic conditions into the model. At the end of this research a validation was performed of the accuracy of the survival method through the time.

Results: The ROC curve and concordance statistics were evaluated on different time points, the survival model shows a consistent high discriminatory power in terms of the AUC over each time horizon.
The results revealed that time dependent ROC curves for the selected years from 1 to 4 and the first year have the largest area under the curve (AUC). The time dependent curve is evaluated at all event times under the 95% pointwise confidence limits of the fitted model, the AUC was on average around 0.8, with the highest values in the first years.

Implications and recommendations: The results are promising for PD estimation in a lifetime perspective. This method is accurate for IFRS9 ECL purposes as time varying internal (portfolio characteristics) and external (macroeconomic) factors can be incorporated. The dynamic model incorporates the variability and changes of the variables from the past up to now.

Originality/value: To date the survival analysis techniques were used mostly for PD estimations but not in a IFRS9 ECL perspective. Given the nature of this method of estimating the remaining lifetime perspective and the inclusion into the model of the macro variables, this model can be considered adequate according to IFRS9. The paper aimed to present their uses for lifetime prediction.

Keywords: survival analysis, Probability of Default (PD), macro variables, Cox regression

1. Introduction

Resulting from the 2008 financial crisis, the financial systems have implemented a way of calculating the expected credit losses, which identified that the delayed recognition of credit losses on loans and other financial instruments was a weakness in the accounting standards. The International Accounting Standards Board (IASB) published the final version of IFRS9 – Financial Instruments in the middle of 2014. This standard offers all the classification and measurement, impairment, and hedging accounting phases of the IASB’s to replace the IAS 39 Financial Recognition and Measurement.

One of the principal changes and challenges across the modelling phases with this standard was the inclusion of the forward-looking approach into the expected credit losses model, having a more timely recognition of loan losses within a single model that is applicable to all financial instruments subject to impairment accounting. For the financial institutions, this new standard became effective from January 2018, taking into consideration that it was available to be on early application bases for the institutions that required it. The measurement of the expected credit losses (ECL) requires three parameters: the probability of default (PD) which estimates the average percentage of facilities or obligors that will incur in a default event; the loss given default (LGD) estimates the percentage of the exposure that the borrower will loss in case of an default event occurs; the exposure at default (EAD) which represents the amount of outstanding at the moment of default, depending on the characteristics of the asset, the EAD could consider a credit conversion factor (CCF). The product of the three parameters estimates the ECL using the following formula:

\[ ECL = EAD \cdot PD \cdot LGD. \]

This paper presents the advantages of using survival analysis – the Cox Proportional Hazard Rate for the estimation of the probability of default (PD). This approach is a dynamic model which can provide an estimate of the PD over a lifetime using the most recent data within each period evaluated. The proposed approach shows the improvement of explanatory power for forecasting default with the ability to incorporate non-default events and the improvement of accuracy with the incorporation of macroeconomics variables. The main advantage of the research is the application of the Cox Proportional Hazard Rate used as a model to predict the default incorporating the time to event and the macroeconomic conditions into the PD model.

The motivation and contribution of this research was to explore a different alternative to derive the Probability of Default within the framework of IFRS9 ECL, the nature of the survival analysis allows to detect the default within the moments up to default, incorporating the macro-economic variables, and estimate the PD on an obligor or loan basis.
2. Literature Review

Up till now the survival analysis techniques were used mostly for PD estimations but not within the IFRS9 ECL perspective and given the nature of the method of estimating the remaining lifetime perspective, and the inclusion into the model of the macro variables, this model can be considered adequate according to IFRS9, hence the paper aimed to present their uses for lifetime prediction. Some examples can be found for European markets (Belloti and Crook, 2009; Ptak-Chmielewska and Matuszyk, 2014; Thomas and Stepanowa, 2002), and for other locations outside Europe (Agarwal and Maheshwari, 2014; Omoga, 2017).

The survival methods applied cover simple nonparametric models such as the Kaplan-Meier life tables but are also more advanced. The parametric methods cover the Accelerated Failure Time Models (Omoga, 2017). The semiparametric approach based on the Cox Proportional Hazard Model is however the most frequently used approach (Omoga, 2017; Ptak-Chmielewska and Matuszyk, 2014). The discrete Hazard Models and the Mixture Cure Models are getting more attention recently (Kalak and Hudson, 2016; Omoga, 2017), as well as machine learning models e.g. Random Survival Models (Fantazzini and Figini, 2009). The evaluation of the applied techniques gave different results with the Cox Proportional Hazard and the Mixture Cure Models performing significantly better than the parametric models and the traditional regression models (Omoga, 2017).

For the IFRS9 perspective, the first studies using the survival approach are also available. Ertan and Gansmann (2015) in their PD model included idiosyncratic firm-specific factors and systematic macroeconomic conditions. They applied a semi-parametric the Cox Proportional Hazard Model to default data and found that the inclusion of macroeconomic covariates in the regression increases explanatory power and improves the regression results. The study presented a good foundation for the implementation of a new model in line with IFRS 9 implementation and accurately estimated PIT PDs.

For the three parameters required to calculate the ECL, the authors focused on the Probability of Default (PD). The new accounting standards specify three-stage approaches, namely stage 1 which determines the initial recognition of non-credit impaired assets that are recognized within the next 12 months; stage 2 allocates assets that show a significant increase in risk, driven by a deterioration of the credit risk since the origination, and for these cases the lifetime ECL is estimated; stage 3 reflects objective evidence of impairment. Under the scope of the PD modelling, stages 1 and 2 are considered as the so-called performing stages. There are some exemptions in the accounting to identify these rules, such as the credit impaired at initial recognition also known as Purchased or Originated Credit-Impaired Financial Assets. These assets receive a different treatment and are evaluated through the lifetime of the ECL.

To identify the uses of the PD model for stages 1 and 2 it was necessary to recognise the type of the evaluated probability, whilst for all new originations or new facilities into the portfolio, the 12-month probability of default was calculated at the initial recognition.

3. Methodology

Survival analysis is part of the statistics whose main objective is to identify the period until one or more events occur.

In this paper the event was defined as the lack of payment from the borrowers, i.e. the so-called default event in credit risk management. The initial approach used in survival analysis (Cox, 1972), is to assume a continuous random variable with probability of density \( f(t) \) and cumulative function as \( F(t) \), having as an output the probability that the event of default occurs within time \( t \). Density function \( f(t) \) is defined as
\[ f(t) = \lim_{\Delta t \to 0} \frac{P(t \leq T \leq t + \Delta t)}{\Delta t}, \]  

(1)

and the survival function

\[ S(t) = P(T \geq t) = 1 - F(t) = \int_t^{\infty} f(x)dx, \]  

(2)

where \( S(t) \) gives the probability of not being in default before time \( t \).

Finally, hazard rate \( h(t) \) is defined as:

\[ h(t) = \lambda(t) = \lim_{\Delta t \to 0} \frac{P(t < T \leq t + \Delta t | T > t)}{\Delta t} = \frac{f(t)}{1-F(t)} = \frac{f(t)}{S(t)}, \]  

(3)

\[ h(t) = \frac{f(t)}{S(t)}. \]  

(4)

Hazard function (3) gives the probability of default in the coming month, given the fact that it is still on the portfolio or non-default (survives) at time \( t \).

### 3.1. The Cox Proportional Hazard Model

The Cox Proportional Hazard Model, or Cox Regression (Cox, 1972), considers a non-parametric estimation of the survival function using the hazard rate. Cox proposed a relation between covariates and the hazard of experiencing an event and the likelihood estimation approach to estimate the parameters of the regression. Hazard of failure \( \lambda(t | x_i) \) can be described as

\[ \lambda(t | x_i) = \lambda_0(t) e^{\beta' x_i}, \]  

(5)

where \( x \) are the predictors' variables or the independent variables of the regression, \( \beta \) – vector of coefficient per each independent variable, \( \lambda_0(t) \) – baseline hazard rate, and \( e^{\beta' x} \) is defined as the hazard ratio.

The estimation of the survival function:\(^1\)

\[ \hat{S}(t | x_i) = \exp(- \int_0^t \lambda_0(u) \exp(\beta' x_i) du). \]  

(6)

Given the survival function, the derivation of the probability of default using the survival probability \( S(t) \) and the relation with hazard rate \( h(t) \) is computed as

\[ PD(s, t) = P(s < T \leq t | T > s) = 1 - \frac{S(t)}{S(s)} = 1 - h(t), \]  

(7)

which means that for high values of the hazard rates the probability of survival will drop faster, and vice versa.

The lifetime \( PD \) is the probability over the remaining life of the loan, this probability can take the form

\[ PD \; TTC = 1 - S(t). \]  

(8)

While the marginal probability is

\[ Marginal \; PD = Lifetime \; PD \; (t+1) - Lifetime \; PD \; (t) = S(t) - S(t+1). \]  

(9)

The conditional marginal probability of default can take the following form

\[ Conditional \; Marginal \; PD = \frac{S(t)-S(t+1)}{S(t)}. \]  

(10)

\(^1\) The survival function of a Cox Proportional Hazard Model computationally can be obtained with the statistical software SAS and the procedure PHREG.
Under IFRS9, the point in time \( PD (PiT) \) could be derived from the cycle probability of default, as proposed by Perederiry (2015), based on the following transformation that incorporates the systematic dependence

\[
PD \ PiT_{i,T_f} = \Phi \left( \Phi^{-1}(PD_{i,T_f}) - \varphi_{t_0} \sqrt{\rho \alpha_1^2 (T_f - T_0)} \right) \frac{1 - \rho \alpha_1^2 (T_f - T_0)}{\sqrt{1 - \rho \alpha_1^2 (T_f - T_0)}},
\]

where the \( PD PiT \) depends directly on the derivation of the probability of default across the evaluated cycles, for each loan \( i \) at given time \( T_f \), the macroeconomic factor \( \varphi \) at time \( t_0 \), systematic correlation coefficient \( \rho \) and autoregressive component \( \alpha_1^2 \) for period \( T_f - T_0 \), all evaluated on a normal distribution.

For survival modelling it is necessary to consider the different times of the appearance of the loan within the time of evaluation in the portfolio, this distinction is known as censoring. Bellini (2019), published the censoring classification as:

- right censoring, a category which considers loans that do not default during the observation period and those that expire without default or defaulted after the end of the observation moment;
- left censoring: facilities that were opened before the observation period;
- defaulted: loans that defaulted during the observation period.

Figure 1 presents the above described mechanisms.

**Fig. 1.** Censoring techniques
Source: own elaboration.

4. Data and Results

The research was carried out on credit default data for a secured mortgages portfolio and covered a period from 2013 to 2021 on a sample of 70% of the overall population from one portfolio (one of EU countries). The period 2013-2020 was used for a development sample and the remaining sample
for 2021 as the out of time sample. The development sample consisted of 1,629,149 observations, considering the economy cycle within the macro-economic variables. The final model selected had ten variables related with the characteristic of the business case, and two macro-economic variables. The model was stratified using the rating labelled from A to M, where A represents good credit quality (below 0.03% PD) and M the lowest rating quality (above 20% PD). The use of rating as a variable to perform the stratification permitted the segregation of the risk, allowing to have heterogeneity in each stratum and a better risk differentiation. The dataset contained the information of the beginning and end of each loan in the portfolio for the whole observation period, which in this study was considered as the right censoring and defaulted for estimating the survival and hazard functions.

From the modelling perspective, the incorporation of a forward-looking perspective estimation was determined across several techniques. The macroeconomic variables that best correlate with the observed default rate\(^2\) are presented below.

**Gross Domestic Product (GDP):** GDP is a widely used macro-economic variable used across the industry, as it correctly reflects the state or direction of the economy in which the business operates, allowing to capture the business and economic cycles. For this variable fewer or low defaults are expected when economic times expands; four quarter annual change GDP in percentages was used (source: Oxford Economics).

**Unemployment Rate (UR):** the unemployment rate correctly reflects the wealth of the economy within the financial system. For countries with good economy, the unemployment rate is expected to be low, and for ‘bad’ economies the ratio could increase – in practice this behaviour needs to be evaluated within the framework of government support which can shift the expected trend. UR in percent of workforce population was used (source: Oxford Economics).

The number of events and censored cases by rating class (strata) are presented in Table 1. The data set consisted of stratified groups per rating grade, where each stratum has its own baseline hazard function.

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Rating</th>
<th>Total</th>
<th>Event</th>
<th>Censored</th>
<th>Percent Censored</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>44,2706</td>
<td>767</td>
<td>44,1939</td>
<td>99.83</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>37,9114</td>
<td>1,592</td>
<td>37,7522</td>
<td>99.58</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>317,988</td>
<td>2,079</td>
<td>31,5909</td>
<td>99.35</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>266,660</td>
<td>3,334</td>
<td>263,326</td>
<td>98.75</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>141,168</td>
<td>3,635</td>
<td>137,533</td>
<td>97.43</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>36,250</td>
<td>1,554</td>
<td>34,696</td>
<td>95.71</td>
</tr>
<tr>
<td>7</td>
<td>G</td>
<td>19,660</td>
<td>1,308</td>
<td>18,352</td>
<td>93.35</td>
</tr>
<tr>
<td>8</td>
<td>H</td>
<td>11,156</td>
<td>1,219</td>
<td>9,937</td>
<td>89.07</td>
</tr>
<tr>
<td>9</td>
<td>I</td>
<td>7,386</td>
<td>1,369</td>
<td>6,017</td>
<td>81.46</td>
</tr>
<tr>
<td>10</td>
<td>J</td>
<td>3,424</td>
<td>1,057</td>
<td>2,367</td>
<td>69.13</td>
</tr>
<tr>
<td>11</td>
<td>K</td>
<td>1,693</td>
<td>736</td>
<td>957</td>
<td>56.53</td>
</tr>
<tr>
<td>12</td>
<td>L</td>
<td>1,152</td>
<td>702</td>
<td>450</td>
<td>39.06</td>
</tr>
<tr>
<td>13</td>
<td>M</td>
<td>792</td>
<td>619</td>
<td>173</td>
<td>21.84</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1,629,149</td>
<td>19,971</td>
<td>1,609,178</td>
<td>98.77</td>
</tr>
</tbody>
</table>

Source: own elaboration with SAS.

\(^2\) The survival estimation could be prepared with the procedure PROC PHREG and can be requested by the baseline statement in the procedure. This procedure provides two alternative survival estimators in proc phreg: the product-limit and the empirical cumulative hazard.
It was found that the ratio of hazard rate for any two groups was constant over time, the assumption was evaluated assessing the proportional hazard’s function using the Schoenfeld residuals. It was concluded that the evaluated factors have a constant impact on the hazard or risk over the time.

The general model fit statistics are shown in Table 2. The global hypothesis indicates that at least one variable is significant. In Table 2 the authors would like to highlight that the model fit statistic, typically used for model selection and comparison, displays the model fit of a model with no predictors. The results show that the predictors improve the fit of the model as the three displayed statistics criteria decrease. Additionally, testing the global null hypothesis confirms that all the coefficients in the model are 0. This means that performing an overall test of the whole model can predict any change in the hazard rate. The test showed that the Likelihood Ratio, Score and Wald of the regression coefficient were significantly different from 0.

Table 2. Model fit statistics

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Without Covariates</th>
<th>With Covariates</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 LOG L</td>
<td>406,286</td>
<td>405,028</td>
</tr>
<tr>
<td>AIC</td>
<td>406,286</td>
<td>405,052</td>
</tr>
<tr>
<td>SBC</td>
<td>406,286</td>
<td>405,147</td>
</tr>
</tbody>
</table>

Testing Global Null Hypothesis: BETA=0

<table>
<thead>
<tr>
<th>Test</th>
<th>Chi-Square</th>
<th>DF</th>
<th>Pr &gt; ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood Ratio</td>
<td>1,257.75</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Score</td>
<td>1,267.58</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Wald</td>
<td>1,262.05</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

Source: own elaboration with SAS.

The results of the model indicate that all variables covariate is highly significant and with positive hazard ratios. Those hazard ratios, also called incident rates, instantaneous risk, or force of mortality, can be interpreted as the risk of an event of default occurrence for a loan at time \( t \). For the macroeconomics variables, in particular for GDP, for one unit increase \( h(t) = e^{-6.38338} = 0.002 \), the hazard of default goes down by an estimated 99.8%, and for the unemployment rate, one unit the hazard of going into default goes down by an estimated 79.1%. The macroeconomic variables reveal high incidence in the survival of the population (see Table 3). It is worth adding that the effect over the time of the macro variables were considered as a time varying explanatory variables. To include them into the regression and improve the estimation, this was analysed via the split in different quarterly time intervals segregated per bucket of rating, and the proportionality assumption was assessed via the standardised empirical score process. The regression coefficients were the same for all the individuals for all the strata.
Table 3. Analysis of maximum likelihood estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Estimate</th>
<th>DF</th>
<th>Standard Error</th>
<th>Chi-Square</th>
<th>Pr &gt; ChiSq</th>
<th>Hazard Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>-6.38338</td>
<td>1</td>
<td>0.62945</td>
<td>102.8439</td>
<td>&lt;.0001</td>
<td>0.002</td>
</tr>
<tr>
<td>UR</td>
<td>-1.56591</td>
<td>1</td>
<td>0.13238</td>
<td>139.926</td>
<td>&lt;.0001</td>
<td>0.209</td>
</tr>
<tr>
<td>woe_fsi_20_class_imp</td>
<td>-0.00262</td>
<td>1</td>
<td>0.0001169</td>
<td>500.7893</td>
<td>&lt;.0001</td>
<td>0.997</td>
</tr>
<tr>
<td>woe_number_active_cards</td>
<td>-0.00359</td>
<td>1</td>
<td>0.0002201</td>
<td>266.2001</td>
<td>&lt;.0001</td>
<td>0.996</td>
</tr>
<tr>
<td>woe_max_arrears_instalment</td>
<td>-0.00238</td>
<td>1</td>
<td>0.0001325</td>
<td>321.0732</td>
<td>&lt;.0001</td>
<td>0.998</td>
</tr>
<tr>
<td>woe_min_CA_tot_eop_balance</td>
<td>-0.00221</td>
<td>1</td>
<td>0.0001764</td>
<td>156.5088</td>
<td>&lt;.0001</td>
<td>0.998</td>
</tr>
<tr>
<td>woe_breach_past_due_amount</td>
<td>-0.00234</td>
<td>1</td>
<td>0.0001203</td>
<td>378.6927</td>
<td>&lt;.0001</td>
<td>0.998</td>
</tr>
<tr>
<td>woe_bik</td>
<td>-0.00234</td>
<td>1</td>
<td>0.0001484</td>
<td>247.6743</td>
<td>&lt;.0001</td>
<td>0.998</td>
</tr>
<tr>
<td>woe_avg_SA_tot_eop_bal</td>
<td>-0.00412</td>
<td>1</td>
<td>0.0001977</td>
<td>434.8724</td>
<td>&lt;.0001</td>
<td>0.996</td>
</tr>
<tr>
<td>woe_type_of_contract</td>
<td>-0.00047</td>
<td>1</td>
<td>0.0001834</td>
<td>6.5918</td>
<td>0.0102</td>
<td>1.000</td>
</tr>
<tr>
<td>woe_diff_abs_str_rep_date</td>
<td>-0.00204</td>
<td>1</td>
<td>0.0001464</td>
<td>193.1450</td>
<td>&lt;.0001</td>
<td>0.998</td>
</tr>
<tr>
<td>woe_occupation</td>
<td>-0.00278</td>
<td>1</td>
<td>0.0001703</td>
<td>265.8159</td>
<td>&lt;.0001</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Source: own elaboration with SAS.

Figures 2 to 6 present the results of the cumulative hazard and survival function per each rating class, and they show that cumulative hazard rates remains the proportion per each bucket of rating, meaning that non-overlapping per risk was detected. With the survival function curves estimated, the chart shows in the left axis the proportion of the population still non-defaulted, the so-called ‘population live’, and in axis x the time over which the event occurs. Subsequently, as described in the modelling methodology section, the PD lifetime, PD marginal and PD conditional marginal were derived. For the PD lifetime, the probability increases naturally per each rating class. The data for the PD charts were cut up to year 3 for visualisation purposes.

![Cumulative Hazards for Reference Setting](image1)

**Fig. 2.** Cumulative hazard rates by rating stratification

Source: own elaboration with SAS.

![Survivor Functions for Reference Setting](image2)

**Fig. 3.** Survival functions by rating stratification

Source: own elaboration with SAS.
For model evaluation, the ROC curve and AUC measure were used. The ROC curve represents the sensitivity (accuracy of default prediction) and specificity of the model (accuracy of non-default prediction). The AUC (Area Under the ROC curve) measures the discriminatory power of the model. The ROC curve and concordance statistics were evaluated on different time points, the survival model showed a consistent high discriminatory power in terms of the AUC over each time horizon. Differently from the traditional logistic regression models, the survival methods are time-sensitive, and the evaluation needs to be performed on each time-period, Harrell’s’ C index (known as the concordance index) evaluates the goodness-of-fit measure for survival analysis, where data can be censored (see Table 4).

Table 4. Harrell’s concordance statistic

<table>
<thead>
<tr>
<th>Source</th>
<th>Estimate</th>
<th>Concordance</th>
<th>Discordance</th>
<th>Tied in Predictor</th>
<th>Tied in Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.8272</td>
<td>9716369</td>
<td>2027408</td>
<td>5502</td>
<td>8895</td>
</tr>
</tbody>
</table>

Source: own elaboration in SAS.

The results shows the time dependent ROC curves for the selected years from 1 to 4, and these curves were computed by the nearest neighbours technique of Heagerty, Lumley, and Pepe (2000), where the first year had the largest area under the curve (AUC) (see Figure 7).
Fig. 7. ROC curve for survival model – time 1 to 4
Source: own elaboration with SAS.

The time dependent curve was evaluated at all event times under the 95% pointwise confidence limits of the fitted model, the AUC was on average around 0.8, with the highest values for first years (see Figure 8).

Fig. 8. AUC over time for survival model
Source: own elaboration with SAS.
5. Discussion and Conclusions

The purpose of this investigation was to present the advantages of using survival analysis for the estimation of the probability of default (PD), given the particularity of the method within the estimation of the time up to an event occurring. The authors applied the Cox Proportional Hazard Model and the results are promising for PD estimation in a lifetime perspective. The accuracy of such a model was high with the AUC above 0.8 on average. This method is accurate for IFRS9 ECL purposes as time varying internal (portfolio characteristics) and external (macroeconomic) factors can be incorporated. Regarding the dynamic model, it incorporates the variability and changes of the variables throughout the past until now. The Cox Proportional Hazard Model is a suitable model to evaluate continuous covariates, which is not the case for the non-parametric simple Kaplan Meier. For further research it would be beneficial to incorporate randomness using random survival forests.

References


Predykcja niewykonania zobowiązań z wykorzystaniem modelu regresji Coksa i warunków makroekonomicznych – perspektywa czasu życia

Streszczenie

**Cel:** Od czasu wdrożenia Międzynarodowych Standardów Sprawozdawczości Finansowej 9 (MSSF 9), różne techniki estymacji parametrów ryzyka do wyliczenia oczekiwanych strat kredytowych zostały wdrożone w instytucjach finansowych. Celem tego badania jest prezentacja zalet stosowania analizy
przeżycia do estymacji prawdopodobieństwa niewykonania zobowiązań (PD) z wykorzystaniem specyfiki metod estymacji czasu do wystąpienia zdarzenia.

**Metodyka:** Wykorzystano model proporcjonalnych hazardów Coksa jako model do predykcji niewykonania zobowiązań włączający czas do wystąpienia zdarzenia i warunki makroekonomiczne do modelu. W badaniu przeprowadzono walidację metod przeżycia w czasie.

**Wyniki:** Krzywa ROC i statystyki zgodności zostały ocenione dla różnych punktów w czasie. Model przeżycia wykazuje wysoką moc dyskryminacyjną w odniesieniu do AUC dla każdego horyzontu czasowego.

Wyniki pokazują, że zależne od czasu krzywe ROC dla wybranych lat 1-4 i dla pierwszego roku mają najwyższą wartość pola pod krzywą (AUC). Krzywa zależna od czasu jest oceniana dla każdego czasu zdarzenia z 95-procentowym przedziałem ufności estymowanego modelu. Stwierdzono ponadto, że wartość AUC jest średnio na poziomie około 0,8, z najwyższą wartością w pierwszym roku.

**Implikacje i rekomendacje:** Wyniki są obiecujące do estymacji prawdopodobieństwa niewykonania zobowiązań (PD) w perspektywie czasu życia kredytu. Stąd ta metoda jest odpowiednia do szacowania oczekiwnych strat kredytowych w ujęciu MSSF 9, ponieważ zależne od czasu wewnętrzne (charakterystyki portfelowe) i zewnętrzne (makroekonomiczne) czynniki mogą być uwzględnione w modelu. Model dynamiczny uwzględnia zmienność i zmiany charakterystyk historycznie w czasie aż do momentu bieżącego.

**Oryginalność/wartość:** Dotychczas techniki analizy przeżycia były wykorzystywane głównie do estymacji prawdopodobieństwa niewykonania zobowiązań (PD) w perspektywie czasu życia kredytu, ale nie w perspektywie oczekiwnych strat kredytowych w ujęciu MSSF 9. ze względu na specyfikę metod do estymacji pozostałego czasu w perspektywie czasu życia. Dodatkowo włączenie do modelu zmiennych makroekonomicznych jest odpowiednim podejściem w MSSF 9. Artykuł ma na celu prezentację wykorzystania tych metod w predykcji czasu życia kredytu.

**Słowa kluczowe:** analiza przeżycia, prawdopodobieństwo niewykonania zobowiązań, makrozmienne, regresja Coksa