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The Prospect Theory and First Price Auctions: an Explanation of Overbidding

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Abstract: This paper attempted using the prospect theory to explain overbidding in first price auctions. The standard outlook in the literature on auctions is that bidders overbid, but the probability weighting functions are nonlinear as in the prospect theory, so they not only tend to underweight the probabilities of winning the auction but also overweight, so that there are overbidders and underbidders. This paper proves that to some extent, non-linear weighting functions do explain overbidding the risk-neutral Nash equilibrium valuation (RNNE). Furthermore, coherent risk measures, such as certainty equivalent and translation invariance, were used to show loss aversion among bidders, and in line with the prospect theory, convexity was also confirmed with sub-additivity, monotonicity and with positive homogeneity.

Keywords: cumulative prospect theory, first-price auctions, overbidding, probability weighting function (PWF), inverse S-shaped functions.

1. Introduction

The prospect theory was introduced as a critique of the expected utility theory as a decision-making model under risk, in a paper published in *Econometrica* in 1979 by Kahneman and Tversky entitled “Prospect Theory: An Analysis of Decision under Risk”. This paper made a significant contribution since it showed that people are systematically violating the properties of the expected utility model, which at the time was the workhorse model for decision-making under uncertainty. However, the prospect theory has not been applied in economic theory, not because it was irrelevant outside the laboratory setting, but because it was hard to know how to apply it, see Barberis (2013). Mostly, researchers from the behavioural economics, i.e. behavioural finance, are involved with the application of the prospect theory. Studies in finance are conducted through the CAPM models of Sharpe (1964) and Lintner (1965)[1], where investors are evaluating utility in accordance with the expected utility, stating that securities with "higher betas", i.e. the returns of the securities that covary more with the return of the overall market. Although research by Fama, and French (2004) concluded that this model does not receive empirical support[2], the Black (1972) model[3] assumes no riskless asset. This version of the CAPM model[4] is more robust in empirical testing. First, the potential shortcoming of the original CAPM model is that the 'true' market portfolio is unobservable (see: Roll, 1977). Roll (1977) also pointed to mean variance tautology, namely that mean-variance efficiency and the capital asset pricing model equation are mathematically equivalent[5]. Hence, this raised the question of whether one can do better in explaining cross-section average returns using a model in which investors evaluate risk in a psychologically plausible way. Barberis and Huang (2008) studied the asset prices in a one-period economy in which investors derive prospect utility from the change in the values of portfolios. The study found that a security whose return distribution is right tail skewed will be overpriced, relative to an economy with expected utility investors, and will earn a lower average return. Previously, papers used pricing of financial securities when investors make decisions according to the cumulative prospect theory (CPT) of Tversky and Kahneman (1992). Under the cumulative prospect theory, one uses a value function defined over gains and losses, concave over gains and convex over losses, and kinked in the origin, and using weighted probabilities. Furthermore, this study explains that overweighing the

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[1] The Sharpe-Lintner CAPM model is given as $E(R_i) = R_f + \beta_{i,M}\left[E(R_M) - R_f\right]; \beta_M = \frac{cov(R_i,R_M)}{\sigma^2(R_M)}$, where $\beta_{i,M}\left[E(R_M) - R_f\right]$ is premium per unit of beta risk, $E(R_i)$ is the expected return and R_f is a risk free interest rate, see (Fama & French, 2004).

[2] An early test by Fama and MacBeth (1973), Gibbons (1982), and Stambaugh (1982), found that β appears to suffice in explaining expected returns and the sign on the premium on β is +, and in the cross-sectional model of Fama and French (1992) where: regress $\overline{R_\iota} - \overline{R_f} = \gamma + \hat{\beta}_{i,M}\lambda + u_i$. CAPM predictions in the previous model are that $\gamma = 0$ and $\lambda > 0$ by Black (1972). On the other hand, $\lambda = E(R_M - R_f)$ by (Sharpe-Lintner), but the empirical estimates showed that γ was too large and λ too small. Later studies by Fama and French (1992), found that β does not explain cross section average returns, and $+\beta$ premium does not exist in the post-1963 period.

[3] According to Black (1972), $E(R_i) = E(R_z) + \beta_i[E(R_M) - E(R_z)]$, where $E(R_z)$ is the expected return on zero-beta portfolio z. Portfolio z is defined as the portfolio that has the minimum variance of all portfolios uncorrelated with M.

[4] Black (1972) differed from Sharpe-Lintner in $E(R_z)$; in Black $E(R_M) - E(R_z) > 0$ whereas in Sharpe-Lintner $E(R_z) = R_f$. Hence, in either case there is positive premium for β .

[5] See also Pollard (2008), Mean-Variance Efficiency and the Capital Asset Pricing Model, Proof.

tails is a modelling device for capturing the common preference for a lottery-like, positively skewed wealth distribution. Kőszegi and Rabin (2006, 2007, 2009), in their respective papers proposed how to apply the prospect theory in economics, suggesting that the reference point people use to compute gains and losses are their rational expectations and beliefs[6]. In the case of finance, several authors and publications using different techniques to measure the skewness concluded that more positively skewed stocks will have lower on average returns, see: Boyer, Mitton and Vorkink (2010), Bali, Cakici, and Whitelaw (2011). The prospect theory implies that stocks involved in an offering should have lower average returns. Green and Hwang (2012), found that the higher the predicted skewness of an initial public offering stock, the lower its long-term average return. An open IPO, which is a modified Dutch auction (strategically equivalent to a First Price Auction), based on an auction system designed by Wiliam Vickrey[7]. This auction method ranks bids from highest to lowest, then accepts the highest bids that allow all shares to be sold, with all winning bidders paying the same price, similarly to T-bills auction. One consistent outcome found in the experiments involving independent private value first price auctions[8] is that the subjects consistently bid above the risk neutral Nash equilibrium (RNNE) bid, see: Dorsey and Razzolini (2003). For a well-known example of "misbehavior" of bidders in FPA auctions, see: Harrison (1989); Cox, Smith and Walker (1988). Next, the authors examined to what extent the prospect theory is able to explain overbidding in first price auctions (FPA). Research concluded that overbidding occurs when bidders are risk-seeking when faced with a risky choice leading to losses, or risk-averse when faced with a risky choice leading to gains cf. Kirchkamp and Reiss (2004); Kagel and Levin (2002, 2016). Earlier studies explaining the overbidding with CRRA constant relative risk aversion include (Cox et al., 1982, 1983a, 1983b, 1984, 1985). Yet, the earlier literature states that risk aversion cannot be the only factor behind overbidding (cf. Kagel and Roth, 1992). Loss aversion was the main evidence for overbidding in a multidimensional reference dependent model for FPA and SPA (see: Lange, & Ratan 2010). Some studies used the prospect theory in their explanation of overbidding in auctions, examples include (Goeree et al., 2002), that used subject probability weighting function (PWF) suggested by Prelec (1998). Ratan (2009) also used the same PWF, together with a multidimensional reference-dependent model. Armantier and Treich (2009a; 2009b) stated that any star-shaped probability weighting function is able to explain overbidding. I In addition, this paper proves that when overbidding occurs in first price auctions, the results are for symmetric and asymmetric auctions, and also that certainty equivalent function is convex in this case, which implies a risk--seeking utility function.

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[6] For instance, the difference between consumption and derived consumption is utility, where the utility function exhibits loss aversion and diminishing sensitivity.

[7] W. Vickrey's 1996 joint Nobel Prize was in large part awarded for his (1961, 1962) papers which developed some special cases of the Revenue Equivalence Theorem.

[8] The literature on FPA includes Lebrun (1996); Maskin and Riley (2000a); Maskin and Riley (forthcoming); Athey (2001); Lizzeri and Persico (2000).

2. The prospect theory value of a game

Let us consider a game with two possible outcomes: x with probability p and y with probability $1-p$, where $x \geq 0 \geq y$. The prospect theory value of the game is

equation 1

$$V = \pi(p)u(x) + \pi(1-p)u(y)$$

In prospect theory the probability of weighting π is concave and first order convex, e.g.

equation 2

$$\pi^{\beta} = \frac{p^{\beta}}{p^{\beta} + (1-p)^{\beta}}$$

For some $\exists\beta \in (0,1)$. A useful parametrisation of the prospect theory value function is power law function

equation 3

$$u(x) = |x|^{\alpha}\ ; x \geq 0$$

$$u(x) = -\lambda|x|^{\alpha}; x \leq 0$$

A fourfold pattern of risk aversion u is:

1. Risk aversion in the domain of likely gains.
2. Risk aversion in the domain of unlikely gains.
3. Risk seeking in the domain of likely losses.
4. Risk seeking in the domain of unlikely losses.

Some properties of the prospect theory value functions are:

- They are scale invariant, i.e. $\forall k > 0$

Now, let us consider two gambles (uncertain outcomes[9]), the second gamble being scaled by k

equation 4

$$g = \begin{cases} x\ , prob = p \\ y, prob = 1-p \end{cases}$$

equation 5

$$kg = \begin{cases} kx\ , prob = p \\ ky, prob = 1-p \end{cases}$$

and then:

[9] Variance of the variable or squared root of variance.std.deviation, for a continuous variable; or in a context of the CAPM model, the measure is called downside beta that measures downside risk (risk associated with losses).

equation 6

$$V^{PT}(kg) = k^{\alpha} V^{PT}(g)$$

If someone prefers g to g' then they will prefer kg to kg' for $k > 0$, and if $x, y \geq 0$, then:

equation 7

$$V(-g) = -\lambda V(g)$$

If $x', y' \geq 0$, and someone prefers $-g$ to $-g'$, the question that arises is of the robustness of the results, and the results are:
very robust
1. where there is loss aversion at reference point, $\lambda > 1$
medium robust
2. where there is convexity of u for $x < 0$
slightly robust
3. dependent on the underweighting or overweighting the probabilities $\pi(p) \gtrless p$.

In application, often a simplified version of the PT theory is used:

equation 8

$$\pi(p) = p$$

equation 9

$$u(x) = x, for\ x \geq 0$$

equation 10

$$u(x) = \lambda x\ for\ x \leq 0$$

Now, let us consider gamble σ and $-\sigma$ with 50:50 chances. The question arises here as to what risk premium Π would agents pay to avoid small risk σ. It can be proved that as $\sigma \to 0$ this premium becomes $\mathcal{O}(\sigma^2)$, and this is called a second order risk aversion. In fact, it can be shown that for twice continuously differentiable utilities

equation 11

$$\Pi(\sigma) \cong \frac{\rho}{2} \sigma^2$$

where ρ is the curvature of u at 0 that is $\rho = -\frac{u''}{u'}$. Now, let us take an agent with wealth x, and this agent takes the gamble if

equation 12

$$B(\Pi) = \frac{1}{2} u\ (x + \Pi + \sigma) + \frac{1}{2} u\ (x + \Pi - \sigma) \geq u(x)$$

i.e. $\Pi \geq \Pi^*$, where

equation 13

$$B(\Pi^*) = u(x)$$

Now, let us assume that u is twice differentiable and take Taylor expansion of $B(\Pi)$ for small σ and Π:

equation 14

$$u\ (x + \Pi + \sigma) = u(x) + u'(x)(\Pi + \sigma) + \frac{1}{2}u''(x) + (\Pi + \sigma^2) + \mathcal{O}(\Pi + \sigma)^2$$

equation 15

$$u\ (x + \Pi - \sigma) = u(x) + u'(x)(\Pi - \sigma) + \frac{1}{2}u''(x) + (\Pi - \sigma^2) + \mathcal{O}(\Pi - \sigma)^2$$

hence

equation 16

$$B(\Pi) = u(x) + u'(x)\Pi + \frac{1}{2}u''(x)[\sigma^2 + \Pi^2] + \mathcal{O}(\sigma^2 + \Pi^2)$$

Now, using the definition of $B(\Pi^*) = u(x)$ to get:

equation 17

$$\Pi^* = \frac{\rho}{2}[\sigma^2 + \Pi^{*2}] + \mathcal{O}(\sigma^2 + \Pi^{*2})$$

and to solve: $\Pi^* = \frac{\rho}{2}[\sigma^2 + \Pi^{*2}]$ for small σ, call $\rho' = \frac{\rho}{2}$. First, find the roots of

equation 18

$$\Pi^{*2} - \frac{1}{\rho'}\Pi^* + \sigma^2 = 0$$

then compute the discriminant

equation 19

$$\Delta = \frac{1}{\rho'^2} - 4\sigma^2$$

thus the roots are

equation 20

$$\Pi^* = \frac{1}{2\rho'} \pm \frac{1}{2}\left(\frac{1}{\rho'^2} - 4\sigma^2\right)^{\frac{1}{2}}$$

As when there is no risk, the risk premium should be 0, then the relevant root is

$$\Pi^* = \frac{1}{2\rho'} - \frac{1}{2}\left(\frac{1}{\rho'^2} - 4\sigma^2\right)^{\frac{1}{2}}$$

Then, take the Taylor expansion for small σ

equation 21

$$\Pi^* = \frac{1}{2\rho'} - \frac{1}{2\rho'}\ (1 - 4\rho'^2\sigma^2)^{\frac{1}{2}} = \frac{1}{2\rho'} - \frac{1}{2\rho'}\left(1 - \frac{1}{2}4\rho'^2\sigma^2 + O(\sigma^2)\right)^{\frac{1}{2}} = \rho'\sigma^2$$

Now, let us remember that $\rho' = \frac{\rho}{2}$, hence $\Pi^* = \frac{\rho}{2}\sigma^2$.

First order risk aversion of prospect theory

Let us consider a gamble as for expected utility. One takes the gamble if $\Pi \geq \Pi^*$, where

equation 22

$\pi\left(\frac{1}{2}\right)u(\Pi^* + \sigma) + \pi(0.5)u(\Pi^* - \sigma) = 0$ to show that in the prospect theory, as $\sigma \to\ 0$, risk premium Π is of order σ when reference wealth $x = 0$; this is called first order aversion. Now let us compute Π for $u(x) = x^\alpha$ and $u = -\lambda|x|^\alpha$. Premium Π at $x = 0$ satisfies:

equation 23

$$0 = \pi\left(\frac{1}{2}\right)(\Pi^* + \sigma)^\alpha + \pi(0.5)(-\lambda)|\sigma + \Pi^*|^\alpha$$

Next, cancel $\pi\left(\frac{1}{2}\right)$ and use the fact that $-\sigma + \Pi^* < 0$ to obtain

equation 24

$$0 = (\sigma + \Pi^*)^\alpha - \lambda(\sigma - \Pi^*)^\alpha \Leftrightarrow (\sigma + \Pi^*)^\alpha = \lambda(\sigma - \Pi^*)^\alpha \Leftrightarrow \sigma + \Pi^* = \lambda^{\frac{1}{\alpha}}[\sigma - \Pi^*]$$

then

equation 25

$$\Pi^* = \frac{\lambda^{\frac{1}{\alpha}} - 1}{\lambda^{\frac{1}{\alpha}} + 1}\sigma = k\sigma$$

where k is equal from the second order risk aversion to

equation 26

$$\Pi = k\sigma^2$$

Then

equation 27

$$k\sigma^2 = \rho'[\sigma^2 + k^2\sigma^4] = \rho'\sigma^2 + \mathcal{O}(\sigma^2) \Rightarrow k = \rho' + \mathcal{O}(1)$$

but empirically k value will be

equation 28

$$\lambda = 2, \alpha \simeq 1, k = \frac{\lambda-\alpha}{\lambda+\alpha} = \frac{2-1}{2+1} = \frac{1}{3}$$

Note that when $\lambda = 1$, the agent is risk neutral and the risk premium is 0. Next, two extensions of the prospect theory are presented. First, both outcomes are positive $0 < y < x$, then

equation 29

$$V = v(y) + \pi(p)\big(v(x) - v(y)\big)$$

For the negative gambles, apply the same formula and $0 > y > x$. Continuous gambles' distribution for expected utility gives

equation 30

$$V = \int_{-\infty}^{+\infty} u(x)f(x)dx$$

The prospect theory gives

$$V = \int_{0}^{+\infty} u(x)f(x)\pi'\big(P(g \geq x)\big)dx + \int_{-\infty}^{0} u(x)f(x)\pi'\big(P(g \leq x)\big)dx$$

Kahneman, Knetsch, and Thaler (1990) showed that in expected utility $WTP = WTA$, or willingness to pay is equal to willingness to accept[10]. Otherwise, as in Sugden (1999), and in Horowitz and McConnell (2003):

equation 31

$$\frac{\partial WTP}{\partial y} \approx 1 - \frac{WTP}{WTA}$$

$\frac{\partial WTP}{\partial y}$ is labelled as the income effect. Horowitz and McConnell (2002) found that WTA is about seven times higher than WTP. Hanemann (1991), showed that the difference between WTP and WTA depends on the ratio of ordinary income elasticity of demand for the good with respect to the Allen-Uzawa elasticity of substitution between the

[10] Willingness to accept is the minimum amount of monetary units that a person is willing to accept to abandon a good or to put up with something negative, such as pollution.

good and a composite commodity (see: (Hicks & Allen, 1934 a,b; Uzawa, 1962)). The elasticity of substitution can be defined as

equation 32

$$\sigma = \frac{\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}}{f(x_1, x_2)\frac{\partial^2 f}{\partial x_1 \partial x_2}}$$

or, since $x_1 = x_1(p), .., x_n = x_n(p)$, and $\lambda = \lambda(p) = \sum_{i=1}^{n}; p_i x_i(p)$, $\lambda(p)$ is the unit cost function, then

equation 33

$$\sigma_{ij} = \frac{\lambda \frac{\partial x_i}{\partial p_j}}{x_i x_j}$$

and $\frac{\partial x_i}{\partial p_j} = \frac{\partial^2 \lambda}{\partial p_i \partial p_j}$, then

equation 34

$$\sigma_{ij} = \frac{\lambda^2 \frac{\partial^2 \lambda}{\partial p_i \partial p_j}}{\frac{\partial \lambda}{\partial p_i}\frac{\partial \lambda}{\partial p_j}}$$

Now the aggregate Allen-Uzawa elasticity of substitution between consumption denoted by q and the Hicksian composite commodity[11] $x_0 \equiv \sum \overline{p_\iota} x_i$, is denoted σ_0. Following Diewert (1974), a formula that relates ξ, which is the income elasticity and $\sigma_{0,}$ the compensated own price elasticity for commodity consumption q or $\varepsilon = -\sigma_0(1-\alpha)$ which is price demand elasticity. The expenditure function is given as $m(p, q, u) \equiv \sum p_i g^i(p, q, u)$, where agents maximise:

equation 35

$$u(x, q) \text{ s.t. } \sum p_i x_i = y \text{ and } x_i = h^i(p, q, y), i = 1, \dots, N$$

The indirect utility function is given as $v(p, q, y) \equiv u[h(p, q, y), q]$. If $q_1 > q_0$, then $u^1 \equiv v(p, q^1, y) \geq 0$. Also when $q = \hat{g}^q(p, \pi, u)$, and inverse compensated demand i.e. WTP is given as $\pi = \hat{\pi}(p, q, u)$, now the two related expenditure function become

[11] Hicks' Composite Commodity Theorem states that "if the prices of a group of goods change in the same proportion, that group of goods behaves just as if it were a single commodity", or "A set of goods whose relative prices do not change, so that they can be treated as a single commodity".

equation 36

$$m(p,q,u) \equiv \hat{m}[p,\hat{\pi}(p,q,u)] - \hat{\pi}(p,q,u) \cdot q \Rightarrow m(p,q,u) \equiv -\hat{\pi}(p,q,u)$$

so no compensating (CV)[12] and equivalent (EV)[13] variations (see Hicks (1939) for price changes) are given as:

equation 37

$$CV = \int_{q_0}^{q_1} \hat{\pi}(p,q,u^0)\,dq\ ;\ EV = \int_{q_0}^{q_1} \hat{\pi}(p,q,u^1)\,dq.$$

Now $\xi = \frac{\partial \ln \hat{\pi}(p,q,u)}{\partial lny}$ is the income elasticity of $\ln \hat{\pi}(p,q,u)$, and by implicit differentiation of $q = \hat{h}^q(p,\pi y + \pi q)$ one obtains

equation 38

$$\frac{\partial \hat{\pi}(p,q,u)}{\partial y} = -\frac{\hat{h}_y^q\,(p,\pi,y+\pi q)}{\hat{h}_\pi^q\,(p,\pi,y+\pi q) + q\hat{h}_y^q(p,\pi,y+\pi q)}$$

Hence, the previous expression can be rewritten as

equation 39

$$\frac{\partial \hat{\pi}(p,q,u)}{\partial y} = -\frac{\hat{h}_y^q\,(p,\pi,y+\pi q)}{\hat{g}_\pi^q[p,\pi,v(p,q,y)]}$$

This expression $\hat{g}_\pi^q[p,\pi,v(p,q,y)] < 0$, so that the whole expression becomes positive. If this is converted into the elasticity form, it will become

equation 40

$$\xi = \frac{\eta(1-\alpha)}{\varepsilon}$$

In the previous expression η income elasticity of the direct ordinary demand is

equation 41

$$\eta \equiv \frac{(y+\pi q)\hat{h}_y^q\,(p,\pi,y+\pi q)}{\hat{h}_q\,(p,\pi,y+\pi q)}$$

and the budget share α of consumption q to adjusted income

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[12] $CV = w - e(p_1,u_0)$, where w initial wealth is $w = p_0, u_0$.
[13] $EV = e(p_0,u_1) - w$, where $w = e(p_1,u_1)$.

equation 42

$$\alpha \equiv \frac{\pi\, \hat{h}^q\, (p, \pi, y + \pi q)}{y + \pi q}$$

thus the own price elasticity of the compensated demand function for q is:

equation 43

$$\varepsilon = \frac{\pi \hat{g}_\pi^q [p, \pi, v(p, q, y)]}{\hat{g}^q [p, \pi, v(p, q, y)]}$$

As previously it was known that $\varepsilon = -\sigma_0(1-\alpha)$, $\xi = \frac{\eta(1-\alpha)}{\varepsilon}$ can be written as $\xi = \frac{\eta}{\sigma_0}$. Hence, the difference between CV and EV (compensating and equivalent variation) depends not only on income effects η but also on substitution effects σ_0. Now, assume some quantity Ⅎ

equation 44

$$\mathbb{A} = \int_{q_0}^{q_1} \hat{\pi}(p, q, y) dq$$

and propose that $CV = EV = \mathbb{A}$ so that

equation 45

$$CV = \int_{q_0}^{q_1} \hat{\pi}(p, q, u^0)\, dq = \; EV = \int_{q_0}^{q_1} \hat{\pi}(p, q, u^1)\, dq = \mathbb{A} = \int_{q_0}^{q_1} \hat{\pi}(p, q, y) dq$$

then $CV = EV = \mathbb{A} = 0$. This is the case when $\eta = 0$ and $\sigma_0 = \infty$, so that $\xi = \frac{\eta}{\sigma_0} = 0$ and there would be no substantial difference between CV and EV. However, if $\sigma_0 \to 0$ then if there are very few substitute products of q in x this generates values $\xi \to \infty$. Another association here is

equation 46

$$V = \mathbb{A} - \lambda x\, ; \mathbb{A} - \lambda x \geq 0$$

and $WTP = \frac{\mathbb{A}}{\lambda}$, or

equation 47

$$WTP = \frac{\int_{q_0}^{q_1} \hat{\pi}(p, q, y) dq}{\lambda}; if\ \lambda = 1 \Rightarrow \mathbf{WTP} = \mathbf{WTA}$$

Contributions in the literature include Knez, Smith and Williams (1985), who argued that the difference between buying and selling prices can be attributed by the thoughtless application of normally sensible bargaining habits, namely understanding one's WTP and overstating the minimum acceptable price at which one would like to sell (WTA). Coursey, Hovis and Schultze (1987), stated that the discrepancy between WTP and WTA diminished with the experience of the market setting[14].

2.1. Risk seeking

Let us take stock market return function as $R = \mu + \sigma n$,where $n \sim N(0,1)$ is a return of the 'gambles' or actions with uncertain outcomes. Investors invest in proportion θ in stock and with proportion $1 - \theta$ in a riskless bond with return 0. Total return is then

equation 48

$$\theta R + (1 - \theta)0 = \theta(\mu + \sigma n)$$

Prospect $\pi(t) = p$ is given as

equation 49

$$V = \int_{-\infty}^{+\infty} u\big(\theta(\mu + \sigma n)\big) f(n) dn$$

Since $u = x^{\alpha}$ for positive and $u = -\lambda|x|^{\alpha}$, using the homotheticity[15] one obtains:

equation 50

$$V = \int_{-\infty}^{+\infty} |\theta|^{\alpha} u(\mu + \sigma n) f(n) dn = |\theta|^{\alpha} \int_{-\infty}^{+\infty} u(\mu + \sigma n) f(n) dn$$

Thus, from previous optimal $\theta = [0, \infty]$,depending on the sign of the last integral. This is a problem because one does not have a concave function, and without concavity it is easy to have those instant solutions. One solution here is that $V^{E} + V^{Pt}$ or the expected utility value function added up with the prospect theory value function. Now if one implicitly takes reference point R_t to be the wealth at $t = 0$, then the gamble is $W_0 + g$:

equation 51

$$V^{PT} = V^{PT}(W_0 + g - Rt)$$

Barberis, Huang and Santos (2001) also drew on the prospect theory and proposed a new asset pricing framework that is derived in part by the traditional consumption-based approach (see Lucas, 1978), but also incorporated the prospect theory of

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[14] For a further review on the reasons behind the differences between WTP and WTA see (Kahneman, Knetsch & Thaler, 1991).

[15] Definition: function $v: R^{m} \to R$ is called homothetic if it is a monotonic transformation of a homogenous function, that is there exists a strictly increasing function $g: R \to R$ and a homogenous function $u: R^{n} \to R$ such as $v = g \circ u$ as a composition of functions.

Kahneman and Tversky (1979), and insights by Thaler and Johnson (1991). Now, welfare is hard because it depends on the time frame. Let us take the integrated and separated prospect value functions:

equation 52

$$V^I = V^{PT}(\sum g_i)\ ; V^S = \sum V^{PT}(g_i)$$

The costs of business cycles are given as $c_t = c + \varepsilon_t$,where c is average monthly consumption, with normal iid ε_t with $E\varepsilon_t = 0$, if one takes $R_t = c = 0$. With the prospect value integrated over one year and the segregated one given as

equation 53

$$V^I = V^{PT}(\sum \varepsilon_t) = V^{PT}\left(12^{\frac{1}{2}}\sigma_\varepsilon n_1\right); V^S = 12\sigma_\varepsilon^\alpha V^{PT}(n)$$

In the expected utility, welfare is defined as $V = Eu(c + \varepsilon_t)$, measure of welfare loss due to business cycle is imbedded in ε_t by fraction λ of the consumption that people would accept to give up in order to avoid consumption variability, and λ solves:

equation 54

$$V\ =\ Eu\,(c\ +\ \varepsilon_t)\ =\ u\left((1\ -\ \lambda)c\right)$$

$u(c_t) = \frac{c^{1-\gamma}}{1-\gamma}$ for a positive $\gamma \neq 1$, and to show that $\lambda = \frac{\gamma}{2}\sigma^2 = 2\gamma \cdot 10^{-4}$. If $\gamma \simeq 1$, then consumers in accordance with the prospect theory value more the stability of consumption around the reference point where they are first order risk averse, and their risk aversion depends on their horizon.

2.2. Cumulative prospect theory

In 1992, Tversky and Kahneman proposed a new theory known as the cumulative prospect theory. The prospect similar to the prospect theory is denoted by (x, p) where x-are the outcomes of the prospect and p are their respective probabilities. The reference point is defined as $x_0 = 0$, and all other outcomes are defined in terms of the reference point. A prospect with $n + m + 1$ outcomes is given by $(x_{-m}, p_{-m}; \dots \dots ; . x_m; p_m)$, where $n, m \geq 0$, and $x_{-m} \leq \cdots .. \leq x_m$. Now, to denote a prospect with f and f^+, which is the positive part of the prospect $(x_1, p_1, \dots, x_n, p_n)$, and f^- is the non-positive part of the prospect $(x_{-m}, p_{-m}, \dots ., x_0, p_0)$, then the value of the prospect is given as

equation 55

$$V(x,p) = V(f) = V^-(f^-)V^+(f^+)$$

which is separated in terms of gains and losses. If all of the outcomes in the prospect are positive, then $V(f) = V^{+}(f^{+})$, and if all the outcomes are negative, then $V(f) = V^{-}(f^{-})$. The values of the positive and negative outcomes are given as:

equation 56

$$V^{+}(f^{+}) = \sum_{i=1}^{n} \pi_i^{+} c(x_i); V^{-}(f^{-}) = \sum_{i=1}^{n} \pi_i^{-} c(x_i)$$

It is assumed that there exists a strictly increasing value function that satisfies $v: x \to \mathbb{R}$, satisfying $v(x_0) = v(0) = 0$ and $\pi^{+}(f^{+}) = (\pi_1^{+}, \ldots., \pi_n^{+})$, and $\pi^{-}(f^{-}) = (\pi_{-m}^{-}, \ldots., 0)$, the decision weights or the probabilistic distortions for gains are given as:

equation 57

$$\pi_n^{+} = w^{+}(p_n); \pi_i^{+} = w^{+}(p_i + \cdots + p_n) - w^{+}(p_{i+1} + \cdots + p_n),$$
$$\forall i: (0 \le i \le n-1) = w^{+}\left(\sum_{j=1}^{n} p_j\right) - w^{+}\left(\sum_{j=i+1}^{n} p_j\right)$$

Similarly, the decision weights for losses are given as:

$$\pi_n^{-} = w^{-}(p_{-m}); \pi_i^{-} = w^{-+}(p_{-m} + \cdots + p_{ni}) -$$
$$w^{-}(p_{-m} + \cdots + p_{i-1}), \forall i: (1 - m \le i \le 0) = w^{-}\left(\sum_{j=-m}^{i} p_j\right) - w^{-}\left(\sum_{j=-m}^{i-1} p_j\right)$$

where $w^{+}(0) = w^{-}(0) = 0$ and $w^{+}(1) = w^{-}(1) = 1$, because if something is impossible it should not impact on individual preferences, and when something is certain to happen, then the effect should be the value that the outcome is given. Hence, π_i are decision weights or probability distortion functions, and w^{+}, w^{-} are the decision weighting functions. Therefore, now the value of a prospect is given as:

equation 58

$$V(x,p) = w(p_{-m})v(x_{-m}) + \sum_{i=-m+1}^{0}\left[w^{-}\left(\sum_{j=-m}^{i} p_j\right) - w^{-}\left(\sum_{j=-m}^{i-1} p_j\right)\right]v(x_i)$$
$$+ w(p_n)v(x_n) + \sum_{i=1}^{n-1}\left[w^{+}\left(\sum_{j=-m}^{n} p_j\right) - w^{+}\left(\sum_{j=-m}^{n} p_j\right)\right]v(x_i)$$

Prospect $(x,p)$is more preferred than (y,q) so that $V(x,p) > V(y,q)$ and is indifferent when $V(x,p) = V(y,q)$. Thus, for decisions under risk one has:

equation 59

$$V = V^{+}(f^{+}) + V^{-}(f^{-}) = \sum_{i=-m}^{0} \pi_i^{-} v(x_i) + \sum_{i=1}^{n} \pi_i^{+} v(x_i)$$

weighting functions for gains w^{+} and losses w^{-} can be defined as mapping that assign event A_i to a space denoted as S ,as a number between 0 and 1. The previous satisfy $W^{-}(\emptyset) = W^{+}(\emptyset) = 0$ and $W^{+}(S) = W^{-}(S) = 1$, and $W^{+}(A_i) \leq W^{+}(A_j)$; $W^{-}(A_i) \leq W^{-}(A_j)$; $\exists A_{ij}: A_i \supset A_j$ where

1. $\pi_n^{+} = W^{+}(A_n)$
2. $\pi_i^{+} = W^{+}(A_i \cup A_{i+1} \cup \ldots . \cup A_n) - W^{+}(A_{i+1} \cup A_n), \forall i: 0 \leq i \leq (n-1)$
3. $\pi_{-m}^{-} = W^{-}(A_{-m})$
4. $\pi_i^{-} = W^{-}(A_{-m} \cup A_{i+1} \cup \ldots . \cup A_i) - W^{-}(A_{-m} \cup A_{i-1}), \forall: (1-m) \leq i \leq 0.$

The decision weighting functions W^{+} and W^{-} are defined by the previously mentioned properties above, but they are not directly observable (see: (Wakker & Tversky, 1993)). Therefore, a two-stage decision process is assumed, following: Tversky and Fox (1995); Fox et al. (1996); Kilka and Weber (2001). Fox and Tversky (1998) proposed specifying the weighing function by a two-stage approach:

equation 60

$$W(A_i) = w_R\big(q(A_i)\big)$$

where W is a weighting function, q are probability judgments following the support theory (see Tversky and Koehler (1994)), A_i is the event considered, and w_R is the probability weighting function under risk. Wakker (2001) gave a formal justification for this decomposition of the weighting function. The axiomisation of the cumulative prospect theory (CPT) was presented in (Wakker & Tversky, 1993; Chateauneuf & Wakker, 1999). The value function exhibits the same properties as fin the prospect theory, i.e. reference dependence, diminishing sensitivity, and loss aversion. Hence, $v(x)$ is concave above the reference point$v'' \leq 0; x \geq 0$, and convex above the reference point $v'' \geq 0; x \leq 0$. The previous reflects diminishing sensitivity, which means the impact of changes in the domain of gains and losses diminishes when the distance from the reference point increases. The value function is steeper for losses than for gains, i.e. $v'(x) < v'(-x); x \geq 0$, since losses persist longer than gains, there is also extensive experimental evidence that losses have greater impact on preferences than gains (cf. Tversky & Kahneman, 1991). The parametric form of the value function proposed by Tversky and Kahneman (1992) is

equation 61

$$v(x) = \begin{cases} x^{\alpha}\ if\ x \geq 0 \\ -\lambda(-x)^{-\beta}\ if\ x \leq 0 \end{cases}$$

The decision-weighting function takes cumulative probabilities and weights them nonlinearly. This means that a change in probability from 0.1 to 0 and from 0.9 to 1 has more influence than a change in the probability from 0.4 to 0.5. Small probabilities tend to be overweighted, and moderate to high probabilities tend to be underweighted. This give rise to an S-shaped function concave near 0, and convex near 1 (inverse S-shaped). Function $w(p)$ exhibits subadditivity[16] if $\exists\epsilon_1; \exists\epsilon_2$, such that

equation 62

$$\begin{cases} w(q) \leq w(p+q) - w(p); \forall(p+q): p+q \leq 1-\epsilon_1 \\ 1 - w(1-q) \geq w(p+q) - w(p), \forall p: p \geq \epsilon_2 \end{cases}$$

The following two equations give the parametric form proposed by Tversky and Kahneman (1992)

equation 63

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}; w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}$$

The value function where loss aversion parameter λ is given as

equation 64

$$v(x) = \begin{cases} f(x)\ if\ x > 0; \\ 0\ if\ x = 0; \\ \lambda \cdot g(x)\ if\ x < 0; \end{cases}$$

where $f(x)$ and $g(x)$ are defined as follows:

equation 65

$$f(x) = \begin{cases} x^\alpha\ if\ \alpha > 0 \\ \ln(x)\ if\ \alpha = 0 \\ 1-(1+x)^\alpha\ if\ \alpha < 0 \end{cases}; g(x) = \begin{cases} -(x)^{-\beta}\ if\ \beta > 0 \\ -\ln(-x)\ if\ \beta = 0 \\ (1-x)^{-\beta} - 1\ if\ \beta < 0 \end{cases}$$

Another parametric form[17] which serves as a weighing function was proposed by Gonzalez and Wu (1999)

equation 66

$$w(p) = \frac{\eta p^\gamma}{\eta p^\gamma + (1-p)^\gamma}$$

The curvature parameter is the previous function γ which represents the degree of diminishing sensitivity, the degree of curvature increases as $\gamma \gg 0$ increases.

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[16] Subadditivity: a subadditive function is a $f: A \to B$ where A is a domain, and B is an ordered co-domain that are closed following the property: $\forall x, y \in A, f(x+y) \leq f(x) + f(y)$.

[17] That exhibits properties consistent with the principles of the cumulative prospect theory, more specifically the inverse S-shape.

The elevation parameter represents the attractiveness of the bet and the elevation increases as $\eta \gg 0$ or $\eta \to \infty$. Another weighting function was proposed by Prelec (1998)

equation 67

$$w(p) = e^{(-logp)^{\gamma}}$$

$\gamma > 0$ and the point at which the weighting function crosses the line $w(p) = p$ is fixed at $\frac{1}{e} \approx 0.36788$. The inflection point for the parametric equations typically occurs at $p < 0.4$. The function exhibits the inverse s-shape, concave for lower probabilities and convex for the upper probabilities.

2.3. Methods of estimation

The methods described here include local search optimisation and the nonlinear squares approach. The search space for both parameters (alpha and gamma) was restricted between 0 and 1. With a small search space, it is easier to find the parameters that give the smallest MSE (mean-squared error). With the local search optimisation one can constrain the values of the parameters.

2.3.1. Nonlinear least squares

This method follows the method introduced by Scales (1985), and Aster et al. (2005). It is known to minimise the weighted sum of the square of the residual.

equation 68

$$\varepsilon_i(\theta) = V(x_i, p; \theta) - V(C; \theta), i = 1 \dots n$$

$\varepsilon_i(\theta)$ is the residual, while θ is the invested share (vector of parameters to be estimated), $V(C;\theta)$ is the value of certainty equivalent (CE)[18] for the prospect, and $x_i = h^i(p,q,y)$, is the value of each outcome in the sample. Now, $\mathrm{E}(\theta)$ is the weighted sum of the residuals:

equation 69

$$\mathrm{E}(\theta) = \sum_{i=1}^{n} w_i \varepsilon_i(\theta)^2$$

where w_i is the weight associated with each sample i.If $f(\theta) = \sqrt{w}\varepsilon(\theta)$ where $\varepsilon(\theta) = (\varepsilon_1(\theta), \dots, \varepsilon_2(\theta)^T$, and the weights are given with the following diagonal matrix

[18] A certainty equivalent (CE) of prospect x is outcome p such as $p \sim x$, whereas often the symbol x denotes a constant prospect, see (Wakker, 2010).

matrix 1

$$w = \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix}$$

then one can write

$$\mathrm{E}(\theta) = \varepsilon(\theta)^T w \varepsilon(\theta) = f(\theta)^T f(\theta)$$

The minimum of $\mathrm{E}(\theta)$ is given when the gradient is equal to zero:

$$\nabla \mathrm{E}(\theta) = 2J\partial f(\theta) f(\theta) = 0$$

where $J\partial f(\theta)$ is a Jacobian matrix of $f(\theta)$, and thus one can denote

matrix 2

$$J(\theta) = \partial f(\theta) = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \cdots & \frac{\partial f_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial \theta_1} & \cdots & \frac{\partial f_n}{\partial \theta_n} \end{bmatrix}$$

By Newton's iteration method, the search vector for the k^{th} iteration is given as L_k and θ_k is the value for θ in the k^{th} iteration

equation 70

$$\theta_{k_{t+1}} = \theta_k + L_k$$

Expanding $\nabla \mathrm{E}(\theta)$ by using the Taylor series with respect to θ about $\theta_{k+1} = \theta_k$ one obtains

equation 71

$$\begin{aligned} \nabla \mathrm{E}(\theta) &= 2J\partial f(\theta) f(\theta) \\ &= 2\partial f(\theta_k)^T f(\theta_k) + 2\left[\sum_{i=1}^{n} f_i(\theta_k)\partial^2 f_i(\theta_k) + \partial f(\theta_k)^T \partial d f(\theta_k)\right] L_k \end{aligned}$$

where $\partial^2 f_i(\theta)$ is the Hessian matrix of $f_i(\theta)$ which is given by

matrix 3

$$H(\theta) = \partial^2 f(\theta) = \begin{bmatrix} \frac{\partial^2 f_1}{\partial \theta_1^2} & \cdots & \frac{\partial^2 f_1}{\partial \theta_1 \partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f_n}{\partial \theta_n \partial \theta_1} & \cdots & \frac{\partial^2 f_1}{\partial \theta_1^2} \end{bmatrix}$$

Newtons' iteration method (see e.g. (Amparo et al., 2007)), in general is given as

equation 72

$$x_{n+1} = x_n - \frac{f'(n)}{f(n)}$$

Now if:

equation 73

$$H(\theta_k) = \sum_{i=1}^{n} f_i(\theta_k)\partial^2 f_i(\theta_k)$$

the value function for any given lottery is given as

equation 74

$$U(F|v, x_0, w) = \int u(x|x_0)dG(x|x_0)$$
$$= \int_{x<x_0} v(x - x_0)d(1 - w(1 - F(x)) + \int_{x>x_0} v(x - x_0)dw(F(x))$$

where x_0 is the reference point. The consequences above x_0 are considered gains, and ones below are losses. In the previous expression w is a probability weighting function, and F is any given lottery. The cumulative density functions for losses are given as

equation 75

$$G(x|x_0) = 1 - w(1 - F(x)) \text{ for } x \leq x_0$$

and the corresponding cumulative density function for gains is given as

equation 76

$$G(x|x_0) = w(F(x)) \text{ for } x \geq x_0$$

The agent is evaluating the consequences, having in mind the reference-dependent utility function

equation 77

$$u(x|x_0) = v(x - x_0)$$

The probability weighting function is given as $w(p) = e^{-(-lnp)^{\alpha}}$, just as in the Allais paradox (see: Allais (1953)), weighted utility theory is given as

equation 78

$$W(p) = \sum_{x \in C} w(x|p, g)u(x)$$

in the previous expression

equation 79

$$w(x|p,g) = \frac{g(x)p(x)}{\sum_{y \in C} p(y)g(y)}$$

for some function $g: C \to \mathbb{R}$. The probabilities must be distorted if one wants to apply the Allais paradox. One prominent theory that distorts the probabilities to this end is the rank-dependent expected utility theory. The new distorted cumulative distribution function (CDF) is given as $w \circ F$, and then the resulting value function is given as:

equation 80

$$v(x|w) = \int u(x)dw\big(F(x)\big)$$

3. Results from the MATLAB simulation

The results from the MATLAB simulation using the code by Pitcher (2008) are shown below (Figure 1).

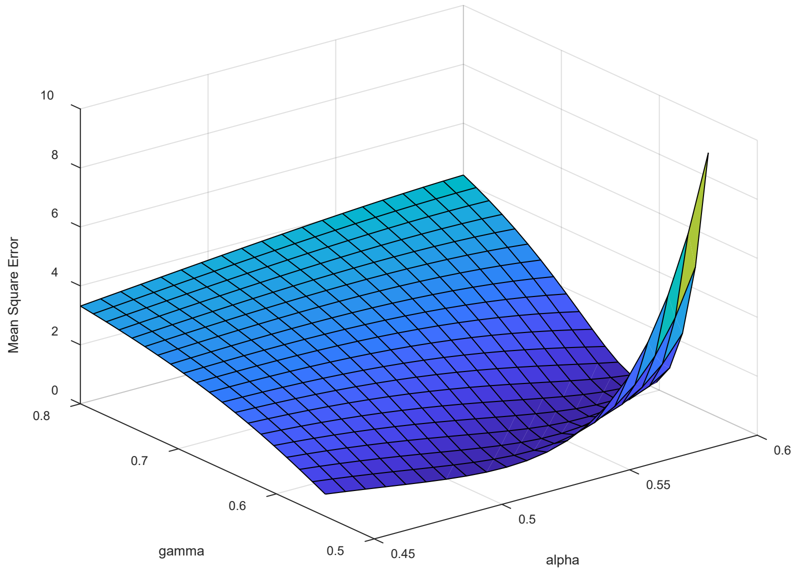

Fig. 1. The values of α and γ that minimise mean squared error (MSE)

Source: author's own calculation.

Table 1. The values of α and γ that minimise mean squared error (MSE)

α	γ	θ	mean MSE	Value function
0.5211	0.5500	[0.2376;0.2565];	0.0049	
$\alpha = \mathrm{A}(I1(I2))$	$\gamma = \Gamma(I2)$	$\theta_1 = \alpha; \theta_2 = \gamma$	$E = \sum(b * ((V - C)^2));$	$V = v * w$

Source: author's own calculation.

where:

Table 2. Supplement

A	Γ	$I1$	$I2$	V	C
0.45	0.55	10	1	2.30	5
0.457895	0.563158	12		2.37	5
0.465789	0.576316	14		2.27	5
0.473684	0.589474	16		2.34	5
0.481579	0.602632	18		2.26	5
0.489474	0.615789	20		2.61	5
0.497368	0.628947	20		2.53	5
0.505263	0.642105	20			
0.513158	0.655263	20			
0.521053	0.668421	20			
0.528947	0.681579	20			
0.536842	0.694737	20			
0.544737	0.707895	20			
0.552632	0.721053	20			
0.560526	0.734211	20			
0.568421	0.747368	1			
0.576316	0.760526	1			
0.584211	0.773684	1			
0.592105	0.786842	1			
0.6	0.8	1			

Source: author's own calculation.

Table 3. Matrix 7X7

0.598684	0	0	0	0	0	0
0	0.49730456	0	0	0	0	0
0	0	0.40161	0	0	0	0
0	0	0	0.343918	0	0	0
0	0	0	0	0.254	0	0
0	0	0	0	0	0.158397	0
0	0	0	0	0	0	0.158397

Source: author's own calculation.

Table 4. v in value function

48.89	23.54	11.96	7.49	3.9375	3.3396	2.0376
50.31	24.73	12.04	7.493	3.9375	3.3396	2.0376
49.19	22.17	11.85	7.49	3.9375	3.3396	2.0376
49.99	23.99	11.96	7.493	3.9375	3.3396	2.0376
48.8	22.59	11.80	7.49	3.9375	3.3396	2.0376
57.16	24.36	12.52	7.493	3.9375	3.3396	2.0376
52.58	26.31	12.64	7.49	3.9375	3.3396	2.0376

Source: author's own calculation.

The previous matrix is multiplied with the weights scalar so to obtain the value function:

$$w = \begin{bmatrix} 0.0139 \\ 0.0082 \\ 0.0264 \\ 0.0128 \\ 0.0309 \\ 0.0077 \\ 0.0102 \end{bmatrix}$$

As for the weight functions (see (Currim & Sarin, 1989)), proposed by Prelec (1998), defined as $w: [0,1] \rightarrow [0,1], \forall: p \in [0,1]$

equation 81

$$w(p) = exp(-b(-lnp)^{\alpha})$$

Ratan (2009) assumed that $\alpha = 1$, thus the previous expression becomes $w(p) = p^b$, and he also claimed that $b > 1$, or in this case:

equation 82

$$w(p) = \exp\left(-(\log(p))^{\gamma}\right.$$

Value function $V: R \rightarrow \mathbb{R}; V; \forall: x \in R$:

equation 83

$$V = \begin{cases} x^{\theta}, x \geq 0 \\ -\lambda(-x)^{\theta}, x < 0 \end{cases}$$

where $\lambda > 1$ and $\theta \in [0,1]$. Now, let us switch to First Price Auctions (FPA).

4. First Price Auctions and high bidding (overbidding)

The bidding strategy in First Price Auctions (FPA)[19] is given as:

equation 84

$$b(v_i) = v - \frac{1}{F^{n-1}(v)} \int_r^v F^{n-1}(s)\, ds$$

$b_0 < v < 1$ or zero otherwise. In the previous integral *s* is a signal, expression $b(v_i)$ represents buyers *i* valuation of the object bid that wins the object, and in such case v_i represents bidders *i* reservation value. In auctions, each bidder calculates his/her winning probability by compounding the probabilities that every other bidder bids less than his/her bid. In equilibrium, any bidder $i \in N$ with valuation v_i has expected payoff:

equation 85

$$\mathrm{E}(p, b_i, v_i) = (v_i - b_i) \frac{(b_i)^{n-1}}{\left(\frac{n}{n-1}\right)^{n-1}}$$

when the CRRA coefficient is being set, i.e. when bidders are not risk neutral, the corresponding expected payoffs (and corresponding revenue) of the bidders are given as:

equation 86

$$\mathrm{E}(R, b_i, v_i) = v_i * \frac{(b_i)^{n-1}}{\left(\frac{n-1}{n-1+a}\right)^{n-1}}$$

If α is CRRA coefficient then:

equation 87

$$\mathrm{E}(R, b_i, v_i) = v_i * \frac{(b_i)^{n-1}}{\alpha^{n-1}}$$

[19] One assumes that CDF or F is a uniform distribution over [0,1].

The generalised FPA-with reserve price bid is given as (cf. (Krishna, 2009)):

equation 88

$$\beta(v) = x - \int_0^v \frac{1 - F(x)}{F(x)} dy$$

In the previous expression x signals are drawn from private values distribution v, so $x_i = v_i$. In the CRRA case utility is given as $U(c) = c^{1-\alpha}$, and now the bid function is

$$b(v_i) = v - \frac{1}{F^{\frac{n-1}{1-a}}(v)} \int_r^v F^{\frac{n-1}{1-a}}(s)\, ds$$

In the previous expression $b_0 < v < 1$ or zero otherwise. In the case where the reserve price is set $r > 0$:

equation 89

$$b(v_i) = v - \frac{1}{F^{\frac{n-1}{1-a}}(v)} \int_r^v F^{\frac{n-1}{1-a}}(s)\, ds$$

Now if the coefficient is CARA (constant absolute risk aversion), i.e. if the utility function is given with the following expression $u(c) = 1 - e^{-ac}$ where $\alpha > 0$, then the according bidding function is given as:

equation 90

$$b(v_i) = v + \frac{1}{a} \ln\,(1 - \frac{e^{-av}}{F^{n-1}(v)} \int_{e^{ar}}^{e^{av}} [F(a^{-1} lnw]^{N-1} dw$$

where *w* represents wealth of the bidder, approximated by his/her valuation of the object which is subject to bidding at the auction. The inverse of equilibrium bidding strategy (Maskin & Riley 2000; Fibich & Gavish, 2011) is given as:

equation 91

$$v'_i(b) = \frac{F_i(v_i(b))}{f_i(v_i(b))} = \left[\left(\frac{1}{n-1} \sum_{j=1}^{n} \frac{1}{v_j(b) - b}\right) - \frac{1}{v_i(b) - b}\right], i = 1, \dots, n$$

Bidders submit bids that are solutions to the optimisation problem, as in (Gayle & Richard, 2008):

equation 92

$$\beta\ (v) = arg \max_{u \in (0,\omega_h)} (v-u) \cdot \left[F_i(\lambda_i(u))\right]^{k_i-1} \prod_{j \neq 1} \left[F_j\left(\lambda_j(u)\right)\right]^{k_j}$$

The probabilities of winning are

equation 93

$$p_i(r) = k_i \int_r^{b(\omega_h)} \frac{\ell_i'(v)}{\ell_i(v)} \prod_{j=1}^{n} \left[\ell_j(v)\right]^{k_j} dv$$

In the previous expression $\ell_i(v) = F_i(\lambda_i(v))$, and r represents the reserve price in auction. Expected revenue for the auctioneer is

equation 94

$$E(p, b_i, v_i) = \omega_h - r \prod_{j=1}^{n} \left[F_j(r)\right]^{k_j} - \int_r^{b(\omega_h)} \frac{\ell_i'(v)}{\ell_i(v)} \prod_{j=1}^{n} \left[\ell_j(v)\right]^{k_j} dv$$

and the expected revenue for the bidders i group is

equation 95

$$E_i\ (p, b_i, v_i) = k_i \int_r^{b(\omega_h)} \left[F_i^{-1}(\ell_i(v)) - v\right] \cdot \frac{\ell_i'(v)}{\ell_i(v)} \prod_{j=1}^{n} \left[\ell_j(v)\right]^{k_j} dv$$

In the prospect theoretical approach, bidders weight probabilities, and they evaluate gains of the lottery relative to theory reference point via the value function. Here, the weighting function is given as in (Currim & Sarin, 1989):

equation 96

$$w(p) = \begin{cases} 0;\ p = 0 \\ \mu p + \eta;\ \mathrm{p} \in (0.1) \\ 1;\ p = 1 \end{cases}$$

where $\eta > 0; \mu > 0$, such that $2\eta + \mu < 1$. Hence, in symmetric FPA, for any bidder $i \in N$ that has valuation v_i with expected payoff $b \in (0, v_i)$, and all other bidders bid $j \in N\ \bar{A}\{i\}$ follow a symmetric differentiable strategy $\beta \in B_j$:

equation 97

$w\left[F^{n-1}\left(\beta^{-1}(b)\right)\right](v_i - b)$ with this probability, weights risk-neutral bidders in FPA bid:

Proposition 1

equation 98

$$\beta_i(v_i) = \frac{1}{w\left(v_i^{n-1}\right)} \int_0^{v_i} x \frac{\partial w(x)^{n-1}}{\partial x} dx = v_i - \frac{\int_0^{v_i} w(x)^{n-1} dx}{w(v_i)^{n-1}}$$

when bidders are assumed to weight probabilities before compounding and evaluate payoffs relative to the zero reference point.

Proof of proposition 1

Here, FOC with respect to b is

equation 99

$$(n-1)w(F\left(\beta^{-1}(b)\right)^{n-2} \frac{\partial w(F\left(\beta^{-1}(b)\right)}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) = w(F\left(\beta^{-1}(b)\right)^{n-1}$$

In symmetric equilibrium $b = \beta(v_i)$, hence

equation 100

$$(n-1)\frac{\partial w\left(F(w_i)\right)}{\partial v_i} \frac{1}{\beta'(v_i)}\left(v_i - \beta(v_i)\right) = w\left(F(w_i)\right)$$

which, when arranged yields:

equation 101

$$\frac{\partial w\left(F(w_i)\right)}{\partial v_i} v_i = \frac{1}{n-1} w(F(v_i)\beta'(v_i) + \frac{\partial w\left(F(w_i)\right)}{\partial v_i}\beta(v_i)$$

If one multiplies the whole expression with $(n-1)w\left(F(w_i)\right)^{n-2}$ one obtains

equation 102

$$w(F(v_i)\beta'(v_i) + (n-1)\frac{\partial w\left(F(w_i)\right)}{\partial v_i} w\left(F(v_i)\right)^{n-2}\beta(v_i)$$
$$= (n-1)\frac{\partial w\left(F(w_i)\right)}{\partial v_i} w\left(F(v_i)\right)^{n-2} v_i$$

after which

equation 103

$$\frac{\partial}{\partial v_i}\left(w\left(F(v_i)^{n-1}\beta(v_i)\right) = v_i \frac{\partial w\left(F(w_i)\right)^{n-1}}{\partial v_i}\right.$$

From $\beta_i(v_i) = \frac{1}{w(v_i^{n-1})}\int_0^{v_i} x \frac{\partial w(x)^{n-1}}{\partial x} dx = v_i - \frac{\int_0^{v_i} w(x)^{n-1}dx}{w(v_i)^{n-1}}$ one can see clearly that $\beta_i < v_i; \forall\, v_i \in (0,1)$. By differentiating β_i with respect to v_i one concludes that it is increasing in v_i since

equation 104

$$1 - \frac{w\big(F(w_i)\big)^{2(n-1)} - \frac{\partial w\big(F(w_i)\big)^{n-1}}{\partial v_i}}{w\big(F(w_i)\big)^{2(n-1)}} > 0$$

Now, when bidders are bid-shading or pretending that $v_i \sim z_i$, and this bidder's expected payoff is $\beta(z) < v_i$, then for competitors β

equation 105

$$Ep(z) = w\big(F(z)\big)^{n-1}(v_i - z) + \int_0^z w\big(F(x)\big)^{n-1} dx$$

Next

equation 106

$$Ep(v_i) - Ep(z) = w\big(F(z)\big)^{n-1} - \int_z^{v_i} w\big(F(x)\big)^{n-1} dx$$

and from the assumption that F (CDF) is a uniform distribution. one has:

equation 107

$\beta^*(v_i) = v_i - \frac{\int_0^{v_i} w(x)^{n-1}dx}{w(v_i)^{n-1}}$ ■ To define reference point r

equation 108

$$r(v) = \frac{\psi}{n} v$$

where function r is defined as: $r\colon (0,1) \to (0,1)$. This result applies $\forall v \in (0,1)$, also in the previous expression $\psi \in (0,1)$. Now, for the equilibrium analysis take any bidder $i \in N$, assuming again that any bidder $j \in N\,\bar{A}\{i\}$ bids according to symmetric, differentiable strategy $\beta \in B_j$, the expected payoff of bidder i from bidding bid $b \in (0, v)$ is given as:

equation 109

$$w\big[F\big(\beta^{-1}(b)\big)\big]\lambda_1\big(v_i - b - r(v_i)\big) - w[1 - F(\beta^{-1}(b)]\lambda r v_i$$

where $\lambda_1 = 1$ if $b \leq v_i - r(v_i)$ and is equal to λ if otherwise. Hence, under the previous conditions a new proposition follows.

Proposition 2

A unique risk-neutral symmetric equilibrium in FPA auctions is characterised by

$$\beta^{+}(v_i) = \begin{cases} (1 + \frac{\psi(\lambda-1)}{n}\beta(v_i); \frac{v_i}{\beta(v_i)} > 1 + \frac{\psi\lambda}{n-\lambda} \\ v_i - \frac{\psi}{n}v_i; \frac{0}{w} \end{cases}$$

when bidders are assumed to weight probabilities after compounding and they are evaluating payoffs relative to the reference point.

Proof of proposition 2

Here, FOC with respect to b is

equation 110

$$\frac{\partial w\left[F\left(\beta^{-1}(b)\right)\right]}{\partial \beta^{-1}(b)}\frac{\partial \beta^{-1}(b)}{\partial b}\lambda\left(v_i - b - r(v_i)\right) - \frac{\partial w\left[1 - F\left(\beta^{-1}(b)\right)\right]}{\partial \beta^{-1}(b)}\frac{\partial \beta^{-1}(b)}{\partial b}\lambda r(v_i) = \lambda_1 w\left[F\left(\beta^{-1}(b)\right)\right]$$

The equilibrium bidding strategy is $b = b(v_i)$, and thus

equation 111

$$\frac{\partial w\left(F(v_i)\right)}{\partial v_i}\frac{1}{\beta'(v_i)}\lambda_1\left(v_i - \beta(v_i) - r(v_i)\right) - \frac{\partial w\left(1 - F(v_i)\right)}{\partial v_i}\frac{1}{\beta'(v_i)}\lambda r(v_i) = \lambda_1 w\left(F(v_i)\right)$$

By arranging the previous expression

equation 112

$$\frac{\partial w\left(F(v_i)\right)}{\partial v_i}\lambda_1\left(v_i - r(v_i)\right) - \frac{\partial w\left(1 - F(v_i)\right)}{\partial v_i}\lambda r(v_i) = \lambda_1\left(w(F(v_i)\beta'(v_i) + \frac{\partial w\left(F(v_i)\right)}{\partial v_i}\beta(v_i)\right) = \lambda_1\frac{\partial}{\partial v_i}[w(F(v_i)\beta(v_i)]$$

for the optimal bid one has:

equation 113

$$\beta(v_i) = \frac{1}{w(F(v_i))}\left[\int_0^{v_i} x\frac{\partial w(x)^{n-1}}{\partial x}dx - \int_0^{v_i} r(x)\frac{\partial w(x)^{n-1}}{\partial x}dx\right] - \frac{\lambda}{\lambda_1}\frac{1}{w(F(v_i))}\int_0^{v_i} r(x)\frac{1-\partial w(x)^{n-1}}{\partial x}$$

In the previous expression $(x)^{n-1} = F(x)$ and one can replace the reference point r with $r = \frac{\psi}{n}$[20]. Now it can be checked whether equilibrium $\exists b > v_i - r(v_i)$ and whether $\exists b = v_i - r(v_i)$[21]. Then the previous equality becomes inequality

equation 114

$$\beta(v_i) < v_i - \frac{v_i}{n} < v_i - r(v_i)$$

The previous concludes that there is no interior solution. Hence, $\beta(v_i) = v_i - r(v_i)$ if one assumes that $b \geq v_i - r(v_i)$. Now, assuming that $b \leq v_i - r(v_i)$, to check whether $b \leq v_i - r(v_i)$ when $\lambda = 1$. Therefore

equation 115

$$\beta(v_i) = \left(1 + \frac{\psi(\lambda-1)}{n}\right)\frac{1}{w(F(v_i))}\int_0^{v_i} x\frac{\partial w(x)^{n-1}}{\partial x}dx$$

The previous expression for high values of λ and v_i means that $\beta(v_i) > v_i - r(v_i)$. If F is uniform distribution, then there exists equilibrium (cf. Reny (2011). Under certain conditions such as when the player set of strategies is non-empty and closed, the density function should be continuous, the type set should be partially ordered, the strategy set needs to be a compact metric space and a semi-lattice with a closed partial order, and the utility function should be measurable and bounded. This article generalised Athey's (2001) and McAdams' (2003) results on the existence of monotone pure strategy equilibria. According to the previous strategy, bidders are allowed to bid as high as they want so that the utility function is not bounded, which means that the strategy sets are not compact either (see also (Keskin, 2011))[22].

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[20] Corresponding expression would be

$$\beta(v_i) = \left(1 - \frac{\psi}{n}\right)\frac{1}{w(F(v_i))}\int_0^{v_i} x\frac{\partial w(x)^{n-1}}{\partial x}dx - \frac{\lambda}{\lambda_1}\frac{\psi}{n}\frac{1}{w(F(v_i))}\int_0^{v_i} x\frac{\partial w\,(1-\partial w(x)^{n-1})}{\partial x}dx.$$

[21] Although $\lambda = 1$ when $b = v_i - r(v_i)$, then $\lambda = \lambda_1$.

[22] Subset S of a topological space X is compact if for every open cover of S there exists a finite subcover of S. If $C = \{U_\alpha : \alpha \in A\}$ is an indexed family of sets U_α, then C is a cover of X if $X \subseteq \bigcup_{\alpha\in A} U_\alpha$. If one takes that this holds: $\mathcal{A} = \{A \in \mathcal{B} \mid \exists U \in \mathcal{O}: A \subseteq U\}$,where B is a topological basis of X and O is an open cover of X. Here A is a refinement of $\mathcal{O}$, and $\forall A \in \mathcal{A}$, and one select $U_A \in \mathcal{O}$, then $\mathcal{C} = \{U_A \in \mathcal{O} \mid A \in \mathcal{A}\}$, see (Munkres, 2000).

However, these two properties can be satisfied when one narrows the strategy set to [0,1]. Hence, if F is a uniform distribution then proposition 2 holds. ■

The definitions and proof of stochastic dominance and overbidding are given in Appendix.

5. Error return function (erf) for the prospect theory

There exists a market with N-number of securities. At some point in time t an investor can purchase α_n units of generic n^{th} security[23]. The allocation is represented by the N^{th} dimensional vector α, where P_t is a price at the generic time t of the generic n^{th} security. Now, given this and with allocation vector α, the investor forms portfolio

equation 116

$$w_t(\alpha) = \alpha' p_t$$

In investment horizon τ, the market prices of the securities are multivariate random variables. The simple function of a one-dimensional random variable price is

equation 117

$$w_{t+\tau}(\alpha) = \alpha' p_{t+\tau}$$

The objective of the investor is ψ , as he/she wants the largest possible amount of benefits according to the non-satiation principle. Absolute wealth is given as

equation 118

$$\psi_\alpha = w_{t+\tau}(\alpha) = \alpha' P_{t+\tau}$$

In relative wealth, the investor is concerned with overperforming a reference portfolio, whose allocation is denote with β, and the objective of maximisation is

equation 119

$$\psi_\alpha \equiv w_{t+\tau}(\alpha) - \gamma(\alpha) w_{t+\tau}(\beta)$$

In the previous equation, γ is a normalisation factor which means that at the time when the investment decision is made, the reference portfolio and the allocation have the same value

equation 120

$$\gamma(\alpha) = \frac{w_t(\alpha)}{w_t(\beta)}$$

[23] In the case of equities these units are shares, in the case of futures these units are contracts.

According to the prospect theory (Kahneman & Tversky, 1979), investors are concerned with changes in wealth rather than in absolute wealth. Thus, their objective becomes

equation 121

$$\psi_\alpha = w_{t+\tau}(\alpha) - w(\alpha)$$

The explicit expression in terms of allocation becomes: $\psi_\alpha = \alpha'(p_{t+\tau} - p_t)$.With market vector M, the introduced previous expression becomes $\psi_\alpha = \alpha' M$, where $M \equiv a + Bp_{t+\tau}$. Furthermore, $\psi_\alpha = \alpha' K p_{t+\tau}$ where $K \equiv I_n - \frac{p_t \beta\prime}{\beta' p_t}$ where I_n is an identity matrix. Hence, it follows that the allocation function is homogenous of degree one $\psi_{\lambda\alpha} = \lambda\psi_\alpha$, and also is an additive function $\psi_{\alpha+\beta} = \psi_\alpha + \psi_\beta$. When ranking two allocations α, β, the two allocations might not be comparable, therefore all the features of allocation are summarised as $\mathcal{S}: \alpha \mapsto \mathcal{S}(\alpha)$. Now the investor can chose the allocation with the highest degree of satisfaction. This approach is different from the stochastic dominance approach (see: (Ingersoll, 1987; Levy, 1998; Yamai & Yoshiba, 2002))[24]. For the features of satisfaction see also Frittelli, Rosazza and Gianin (2002). The scale invariant index is also known as Sharpe ratio defined as $\mathcal{S}R(\alpha) = \frac{E\{\psi_\alpha\}}{Stdev.\{\psi_\alpha\}}$, which means that high standard deviation is a drawback if the expected utility is positive. Additionally, monotonicity requirements are as follows: $\psi_\alpha \geq \psi_\beta\ ; \forall \mathcal{S} \ \Rightarrow \mathcal{S}(\alpha) \geq \mathcal{S}(\beta)$. For sensibility, i.e. monotonicity, see Artzner, Delbaen, Eber, and Heath (1999). A further requirement is that of positive homogeneity: $\psi_{\lambda\alpha} = \lambda\psi_\alpha, \forall\lambda: \lambda \geq 0$, and additive $\mathcal{S}(\alpha+\beta) \geq \mathcal{S}(\alpha) + \mathcal{S}(\beta)$, or sub-additive $\mathcal{S}(\alpha+\beta) \leq \mathcal{S}(\alpha) + \mathcal{S}(\beta)$. Next, regarding concavity and convexity – an index of satisfaction is said to be concave for $\forall\lambda: \lambda \in (0,1)$ and the following inequality holds:

equation 122

$$\mathcal{S}(\lambda\alpha + (1-\lambda)\beta) \geq \lambda\mathcal{S}(\alpha) + (1-\lambda)\mathcal{S}\beta$$

Similarly, the index of satisfaction is said to be convex for $\forall\lambda: \lambda \in (0,1)$ when the following inequality holds $\mathcal{S}(\lambda\alpha + (1-\lambda)\beta) \leq \lambda\mathcal{S}(\alpha) + (1-\lambda)\mathcal{S}\beta$. On the opposite side of the satisfaction risk premium is dissatisfaction due to the uncertainty of risky allocation $R \equiv \mathcal{S}(b) - \mathcal{S}(b+f)$ where b is risk-free allocation and f is any fair game. Risk aversion is if $R(\alpha) \geq 0$, while risk propensity is $R(\alpha) \leq 0$.

6. Certainty equivalent

Let us consider the expected utility from a given allocation

[24] Strong dominance is $F_{\psi_\alpha - \psi_\beta}(0) \equiv \mathbb{P}\{\psi_\alpha - \psi_\beta \leq 0\} = 0$; this is a strong dominance or order zero dominance, and weak dominance when $F_{\psi_\alpha}(\psi) \leq F_{\psi_\beta}(\psi), \forall\psi: \psi \in (-\infty; +\infty)$, this is also called first order dominance.

equation 123

$$\alpha \mapsto E\{u(\psi_\alpha)\} \equiv \int_{\mathbb{R}} u(\psi) f_{\psi_\alpha}(\psi) d\psi$$

In the previous expression, f_{ψ_α} is a PDF of the objective. The certainty equivalent of an allocation is the risk-free amount of money that would make the investor satisfied as a risky allocation

equation 124

$$\alpha \mapsto CE(\alpha) \equiv -\phi \ln\left(\theta_{\psi_\alpha}\left(\frac{i}{\phi}\right)\right)$$

where ϕ has a dimension of money and it cancels it out. These properties are depicted below (Figure 2):

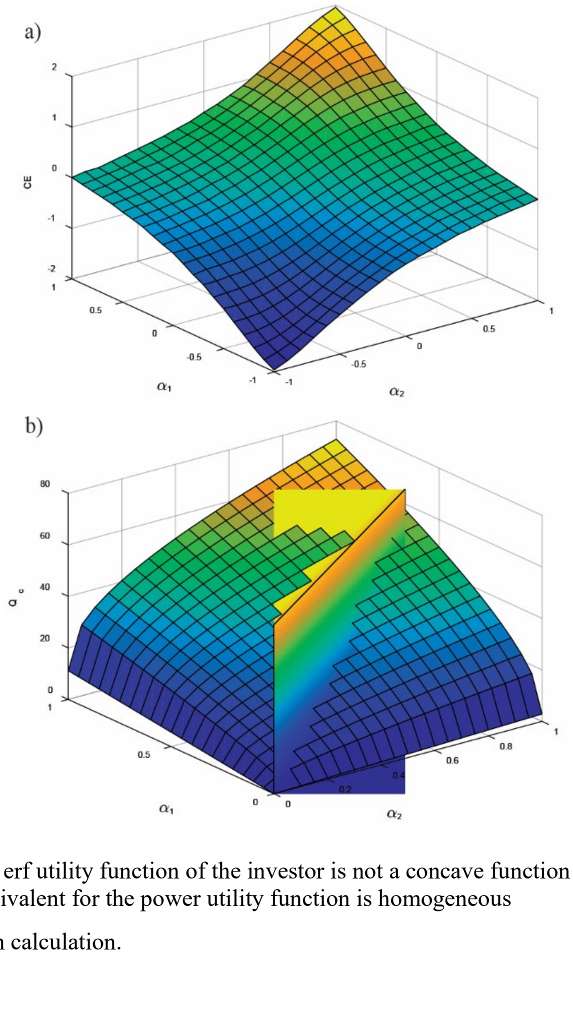

Fig. 2 (a and b). The erf utility function of the investor is not a concave function of allocation, and the certainty equivalent for the power utility function is homogeneous

Source: author's own calculation.

7. Translation invariant

The objective of the investor, besides being positive and homogenous, is additive[25], and by adding two portfolios α, β, one obtains a sum of the two separate objectives

equation 125

$$\psi_{\alpha+\beta} = \psi_\alpha + \psi_\beta$$

The utility from the sum of the two alternatives is unrelated with the satisfaction drawn by the investor from investing in separate portfolios. Thus, the corresponding index of satisfaction is given as:

equation 126

$$\mathcal{S}(\alpha + \beta) = \mathcal{S}(\alpha) + \mathcal{S}(\beta)$$

The previous property is called translation invariance (cf. Meucci, 2005). One can restate the translation invariance property as

equation 127

$$\psi_\beta \equiv 1 \Rightarrow \mathcal{S}(\alpha + \lambda\beta) = \mathcal{S}(\alpha) + \lambda$$

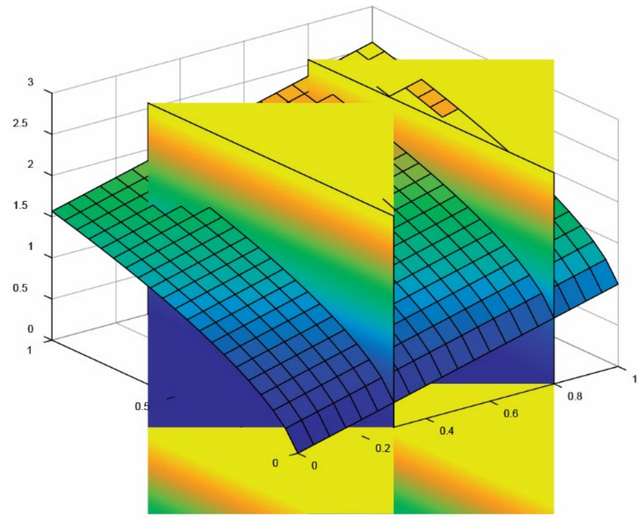

Fig. 3. Translation invariance of the satisfaction index

Source: author's own calculation.

[25] $\mathcal{S}(\alpha + \beta) \geq \mathcal{S}(\alpha) + \mathcal{S}(\beta)$ means super additivity, and $\mathcal{S}(\alpha + \beta) \leq \mathcal{S}(\alpha) + \mathcal{S}(\beta)$ is sub-additivity property.

Now, one can examine the formal definition and proof of translation invariance of a Lebesgue measure[26]. For instance, if $E \in \mathcal{P}(\mathbb{R})$, by adding $\alpha \in \mathbb{R}$ gives as a new set $E+\alpha$ and the Lebesgue outer measure, which is $m^*(E) = m^*(E+\alpha) = m^*\{e\ + a : e \in\ E\ \}$, called the translation invariance of the Lebesgue outer measure.

Theorem 1. Translation invariance of the Lebesgue outer measure: Let $E \in \mathcal{P}(\mathbb{R})$ and $\alpha \in \mathbb{R}$ and then $m^*(E+\alpha) = m^*(E)$.

Proof: Let $(I_n)_{n=1}^{\infty}$ be a sequence of open intervals (an open interval is an interval that does not include endpoints). The open interval $\{x: \alpha < x < \beta\}$ is denoted (α, β) (Gemignani, 1990). Then $(I_n + \alpha\ )_{n=1}^{\infty}$ is a sequence of open intervals that cover $E+\alpha$, and therefore

equation 128

$$m^*(E+\alpha) \leq \sum_{i=1}^{\infty} \ell(I_n+\alpha) = \sum_{n=1}^{\infty} \ell(I_n)$$

Thus, for every sequence of open intervals $(I_n)_{n=1}^{\infty}$ that cover E, one has that $m^*(E+\alpha) \leq \sum_{n=1}^{\infty} \ell(I_n)$ and so $m^*(E+\alpha) \leq m^*(E)$. Let $(I_n)_{n=1}^{\infty}$ be a sequence that covers $E+\alpha$, then $(I_n - \alpha)_{n=1}^{\infty}$ is a sequence of open intervals that cover E, and thus $m^*(E) \leq \sum_{i=1}^{\infty} \ell(I_n - \alpha) = \sum_{n=1}^{\infty} \ell(I_n)$, and no $\forall\ (I_n)_{n=1}^{\infty}$ that cover $E+\alpha$ which gives that $m^*(E) \leq \sum_{n=1}^{\infty} \ell(I_n)$, and now $m^*(E) \leq m^*(E+\alpha)$. From $m^*(E+\alpha) \leq m^*(E)$ and $m^*(E) \leq m^*(E+\alpha)$ one can conclude that $m^*(E) = m^*(E+\alpha)$■.

Theorem 2. Monotonicity of the Lebesgue measure: let $\mathcal{A}, \mathcal{B} \in \mathcal{P}(\mathbb{R})$, and if $\mathcal{A} \subseteq \mathcal{B}$ then $m^*(\mathcal{A}) \leq m^*(\mathcal{B})$.

Proof: If $\{I_n = \alpha_n, \beta_n\}_{n=1}^{\infty}$ such that $\mathcal{B} \subseteq \bigcup_{n=1}^{\infty} I_n$, hence $\mathcal{A} \subseteq \bigcup_{n=1}^{\infty} I_n$ and therefore:

equation 129

$$\left[\sum_{i=1}^{\infty} \ell(I_n): \mathcal{A} \subseteq \bigcup_{i=1}^{\infty} I_n \wedge \{I_n = \alpha_n, \beta_n\}_{n=1}^{\infty}\right] \\ \supseteq \left[\sum_{i=1}^{\infty} \ell(I_n): \mathcal{B} \subseteq \bigcup_{i=1}^{\infty} I_n \wedge \{I_n = \alpha_n, \beta_n\}_{n=1}^{\infty}\right]$$

By the infimum or supremum properties of subsets of sets of real numbers, the previous implies that $m^*(\mathcal{A}) \leq m^*(\mathcal{B})$.

[26] Given open set $\mathcal{S} = \sum_k (\alpha_k, \beta_k)$, then the Lebesgue measure is $\mu_L(\mathcal{S}) = \sum_k (\beta_k - \alpha_k)$, and given a closed set (whose complement is an open set) $\mathcal{S}' \equiv [\alpha, \beta] - \sum_k (\alpha_k - \beta_k)\ \mu_L(\mathcal{S}') = (\beta - \alpha) - \sum_k (\beta_k - \alpha_k)$, see (Kestelman, 1960).

Corollary: $\{\alpha_n\}_{n=1}^{\infty}$ is a sequence of real numbers bounded below, then $\left(\inf_{k\geq n}\{\beta_k\}\right)_{n=1}^{\infty}$ is an increasing sequence.

Proof: Since $\{\alpha_n\}_{n=1}^{\infty}$ is bounded below, then the set $\{\beta_k: k > n\}_{n=1}^{\infty}$is bunded below $\forall n: n \in N$ then:

$$\{\beta_k: k > 1\} \supseteq \{\beta_k: k > 1\} \supseteq \cdots$$

This means that $sup\{\beta_k: k \geq 1\} \leq sup\{\beta_k: k \geq 2\} \leq \cdots$
which follows that $\left(\sup_{k\geq n}\{\beta_k\}\right)_{n=1}^{\infty}$ is an increasing sequence, henceforth $m^*(\mathcal{A}) \leq m^*(\mathcal{B})$ ■.

8. Concluding remarks

This paper attempted to make use of the prospect theory in economics, used by the authors to explain overbidding in first price auctions. This result is documented in the literature of auctions; in particular the phenomenon of overbidding was explained by risk averse bidders. It was proved in that the reference dependence (i.e. the value function depends on gains and losses relative to a status quo and not on final wealth positions as in expected utility theory), loss aversion, and risk seeking are important in explanation of overbidding and the cumulative prospect theory takes all these into account. This paper also proved that in asymmetric first price auctions, overbidding occurs if the reservation price or the item of the object of sale (value) is higher than the bidder's valuation, and also when there exist underbidders since the PWF function is inverse S-shaped and bidders with a low valuation underbid as they are overconfident that they will win the auction. The reason behind underbidding is the overweighting interval of the inverse S-shaped PWF, which is the main reason that PWF cannot sufficiently explain overbidding. In the paper it was shown that the error return function and means squared error are convex, which is in line with the explanations of risk seeking individuals, preferring solutions to avoid losses. Translation invariance and satisfaction (certainty equivalent) were shown that are positive and homogenous, monotonicity was also proven and sub and super additive, which all imply the loss of risk aversion, and, hence overbidding.

Appendix

Stochastic dominance in asymmetric FPA

Definition 1. *First-order stochastic dominance*: $\forall\, F_i\,; \forall F_j$ lotteries (auctions), F stochastically dominates F_j if and only if the agent$F_i \gtrsim F_j$ under weakly increasing utility function u,where $\int u(x)dF_i \geq \int u(x)dF_j$.

Definition 2. *First-order stochastic dominance*: with cumulative distribution functions F_i stochastically dominates F_j if and only if $\forall x: F_i(x) \leq F_j(x)$.

Theorem 3. The previous two definitions are equivalent.

Proof Suppose that $\nexists: \forall x: F_i(x) \leq F_j(x)$, then $\exists x^* \rightarrow F_i(x) > F_j(x)$. Now let us define $u \equiv 1_{\{x>x^*\}}$ by $u(x) = 1$ if $x > x^*$ or otherwise is 0. Thus

equation 130

$$\int u(x)dF_i = 1 - F_i(x^*) < 1 - F_j(x^*) = \int u(x)dF_j$$

Now let us suppose that $\nexists: \int u(x)dF_i \geq \int u(x)dF_j$, in that case

equation 131

$$\int u\big(y(x)\big)dF_i\big(y(x)\big) = \int u\big(y(x)\big)dF_j\big(y(x)\big) \geq \int u(x)dF_j(x)$$

where by equality $y(x) = F_i^{-1}F_j(x)$ and the inequality comes from $u\big(y(x)\big) > u(x); \forall(x)$ because $y(x) \geq x$ and u is weakly increasing ■.

Definition 3. *Second-order stochastic dominance:* $\forall$ F ; $\forall$G lotteries (auctions), F stochastically dominates G if and only if the agent F $\gtrsim$ G under weakly increasing concave utility function u.

Definition 4. $\forall\, F_i; \forall F_j$. F_j is a mean preserving spread of F_j if and only if: $y = x + \epsilon$ for some:$x \sim F_i; y \sim F_j$ and $E(\epsilon|x) = 0$.

Theorem 4: $\int x dF_i = \int y dF_j$, and the following are equivalent.

1. $\int u(x)dF_i(x) \geq \int u(x)dF_j(x)$;
2. F_j is a MPS of F_j;
3. $\forall(v \geq 0): \int_a^v F_j(x)dx \geq \int_a^v F_i(x)dx$

Proof: 2 implies 1. One can write:

equation 132

$$\int u(y)dF_j(y) = \int E[u(x+\epsilon)|x]dF_i(x) \leq \int u[E(x+\epsilon)|x]dF_i(x)$$
$$= \int u(x)dF_i(x)$$

Now to show that 1 is equivalent to 3, define mapping: $\mathcal{J}: \mathbb{R} \rightarrow \mathbb{R}$ by $\mathcal{J}(v) = \int_a^v \big[F_i(x) - F_j(x)\big]$, clearly $\mathcal{J}(a) = 0$; since F_i and F_j have same mean, now by applying integration by parts one obtains

$$\int u(x)d\left(F_i(x) - F_j(x)\right) = \int u''(x)\mathcal{J}(x)dx$$

Hence, the first condition is true if the left hand side is non-negative $\forall(u)$ with $u''(x) \leq 0$. The latter holds if $\mathcal{J}(x) \leq 0$, so that for condition 3 it holds ■.

The relative strength ratio is a possible way of comparing bidder's i beliefs about his/her rival with bidder's j beliefs about the rival is through the relative strength ratio $\mathcal{P}_{ij}$, see (Kirkegaard, 2009):

equation 133

$$\mathcal{P}_{ij} \equiv \frac{F_j(v)}{F_j(v)}; \forall(v): v \in (0, \bar{v})$$

First order stochastic dominance means that the ratio is strongly decreasing, meaning that

equation 134

$$\frac{f_i(v)}{F_i(v)} > \frac{f_j(v)}{F_j(v)}\ ; \forall(v): v \in (0, \bar{v})$$

therefore $F_i(v)$ dominates $F_j(v)$ in terms of invert hazard rate. The comparison of the bidder's payoffs is given as:

equation 135

$$Ru_{i,j}(v) = \frac{(v-b_i(v))\mathcal{H}(b_i(v))p_i^j(v)}{\left(v-b_j(v)\right)\mathcal{H}\left(b_j(v)\right)q_j^i(v)}; Ru_{i,j}(v) = \frac{F_j(v)}{F_i(v)}.$$

where in previous expression, $p_i^j(v)$ and $q_j^i(v)$ are the respective probabilities of winning the auction. The bidders are equally well-off at $\bar{v}$ so that $Ru_{i,j}(v) = 1$. Whereas in the previous expression

equation 136

$$b_i(v) = v - \int_0^1 \frac{p_i^j(x)}{p_i^j(v)}dx; b_j(v) = v - \int_0^1 \frac{q_j^i(x)}{q_j^i(v)}dx$$

Proposition 1. The bid function with the probability weighting is

equation 137

$$b_i^*(v) = v_i - \frac{\int_0^{v_i} w\left(F_i(y)\right)dy}{w(F(v_i)}$$

The result holds if all the bidders subjectively weight probabilities with the same inverse S-shaped PWF.

Proof FOC with respect to $b_i^*(v)$ is:

equation 138

$$\frac{\partial w(F(\beta^{-}(b))}{\partial \beta^{-1}(b)}\frac{\partial \beta^{-1}(b)}{\partial b}(v-b)-w(\beta^{-1}(b))=0$$

In a symmetric equilibrium $b=\beta(v_i)$ and $\beta^{-}(b)=v_i$: and $\frac{\partial}{\partial v_i}\left(wF(v_i)\beta(v_i)\right)=v_i\frac{\partial w\,(F(w_i))}{\partial v_i}$, hence

equation 139

$$\beta^*(v_i)=\frac{1}{w(F(v_i)}\int_0^{v_i} y\frac{\partial w(F(y))}{\partial y}dy=v_i-\frac{\int_0^{v_i} w(F_i(y))dy}{w(F(v_i)} \ \blacksquare$$

Proposition 2. Take into consideration Prelec's (1998) weighting function $w(p)=exp\{-(-\ln p)^{\alpha}\};\alpha\in(0,1)$, and teh bidders' weight probabilities with inverse S-shaped PWF that is

equation 140

$$\int_0^1 e^{-(n-1)^{\alpha}(-lny)^{\alpha}}dy\leq\frac{1}{n}$$

In addition, the following applies regarding the critical valuation $\frac{\int_0^{v^*} e^{-(n-1)^{\alpha}(-lny)^{\alpha}}dy}{e^{-(n-1)^{\alpha}(-lny)^{\alpha}}}=\frac{v^*}{n}$; an inverse S-shaped PWF causes underbidders with low valuations to overestimate their chances of winning the auction by which their response would be to lower their bids. Thus$v_B^*\in(0,1)$ such that $\beta_B^U(v_B^*)=\beta_{RN}^U(v_B^*)$ and any bidder with valuation v underbids if $v<v_B^*$, and the bidder overbids if $v>v_B^*$.

Proof: The bidder with valuation 1 overbids, taking the first derivative β_B^U with respect to α

equation 141

$$(1-n)\int_0^1 e^{(-(n-2)(-lny)^{\alpha})(-lny)^{\alpha}\frac{\partial e^{-(-lny)^{\alpha}}}{\partial\alpha}}dy<0$$

At the maximum valuation $\beta_B^U(1)=\beta_{RN}^U(1)$, when $\alpha=1$, then $\beta_B^U(1)>\beta_{RN}^U(1)$ when $\alpha\in(0,1)$, now the bidder with valuation $\hat{v}(b)$ underbids $\beta_B^U(\hat{v}_B)<\beta_{RN}^U(\hat{v}_B)$, and since both $\beta_B^U;\beta_{RN}^U$ are continuous $\exists v_B^*:\beta_B^U(v^*)=\beta_{RN}^U(v^*)$, then there exists unique v with $\beta_B^U(v)-\beta_{RN}^U(v)=0$; this expression is zero when $v=0$, it is negative when $v=\frac{1}{e}$, and positive when $v=1$, and thus any bidder with valuation v underbids if $v<v_B^*$, and the bidder overbids if $v>v_B^*$ $\blacksquare$ (see also (Keskin, 2015)).

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Teoria perspektywy i aukcje pierwszej ceny: wyjaśnienie przebijania

Streszczenie: W artykule podjęto próbę wykorzystania teorii perspektywy do wyjaśnienia przebijania cen w przetargach pierwszej ceny. Standardowym wynikiem w literaturze związanej z aukcjami jest to, że licytanci przebijają ceny, ale funkcje ważenia prawdopodobieństwa są nieliniowe, jak w teorii perspektywy, więc nie tylko mają tendencję do zaniżania wagi prawdopodobieństw wygrania aukcji, ale także przeważania, tak że są licytanci przebijający i oferenci słabsi. Artykuł ten dowodzi, że do pewnego stopnia nieliniowe funkcje ważenia wyjaśniają zawyżanie neutralnej pod względem ryzyka wyceny równowagi Nasha (RNNE). Ponadto zastosowano spójne miary ryzyka, takie jak ekwiwalent pewności i niezmienniczość translacji, aby wykazać awersję do strat wśród oferentów zgodnie z teorią perspektywy, wypukłość została również potwierdzona subaddytywnością, monotonicznością i dodatnią jednorodnością.

Słowa kluczowe: kumulatywna teoria perspektywy, aukcje pierwszej ceny, przelicytowanie, funkcja ważenia prawdopodobieństwa, odwrotne funkcje S-kształtne.