Argumenta Oeconomica
Year 2024, Vol. 52, No. 1
ISSN 2720-5088

Revisiting the effects of labour, capital and scale-augmenting technological progress on steady-state output

Li Sheng
Faculty of Political Science and Public Administration, Shandong University, China
e-mail: edmundsheng@sdu.edu.cn
ORCID: 0000-0002-2700-9975

Yechang Yin
Faculty of Social Sciences, University of Macau, Macao
e-mail: Yc07306@connect.um.edu.mo
ORCID: 0000-0002-6058-2773

Anning Zhang
Faculty of Social Sciences, University of Macau, Macao
e-mail: yc07322@connect.um.edu.mo
ORCID: 0000-0003-4240-8576

Jiwei Wu
Faculty of Social Sciences, University of Macau, Macao
e-mail: yc17309@um.edu.mo
ORCID: 0000-0002-6730-7176

Quote as: Sheng, L., Yin, Y., Zhang, A., & Wu, J. (2024). Revisiting the effects of labour, capital and scale-augmenting technological progress on steady-state output. *Argumenta Oeconomica*, 1(52), 179-187.

DOI: 10.15611/aoe.2024.1.10

JEL Classification: E22, I26, J24, O33

Abstract: The effects of technical change may not exclusively be labour-augmenting, if it is assumed that linear homogeneity or balanced growth is not applicable. This study identified these effects on the steady-state output growth from labour, capital and scale-augmenting technological progress. The findings imply that a drop in investment-goods prices accelerates output growth but to a lesser extent than capital growth, and increasing (decreasing) returns to scale (IRS/DRS) will lead to faster (slower) per-capita output growth, but the effect is free from constant returns to scale (CRS). It was also found that a higher output elasticity of capital can be beneficial to output growth and that a substantial labour supply is conducive to higher income per capita under IRS but will reduce it if DRS are present, but again, CRS does not occur. Differentiating between labour and capital augmentations in empirical research is challenging, as a direct estimation of the impacts of technical change on output growth is impossible. Thus, an indirect estimation was conducted to measure these impacts as a Solow residual.

Keywords: Solow model, Uzawa's theorem, steady-state growth, factor augmentation, labour augmentation, scale effect

1. Introduction

According to Uzawa's (1961) theorem, the growth rates of capital K_t (investment I_t), consumption C_t, and output Y_t are constant in a steady state (known as steady-state growth at time t after point T) and are the same (i.e. $g_K = g_I = g_C = g_Y$) when facing a balanced growth path (BGP). The restriction of the theorem is obvious, as the assumption of BGP entails a technical change for it to be labour-augmenting, namely the Harrod neutral, as if it cannot be held by any production function (PF), no BGP is produced. A long-standing enigma is why labour augmentation should purely be an attribute of technical change. In the Solow model, growth conclusions can be obtained from the capital per unit of effective labour but not from the effective capital per unit of labour (as if so, any results regarding steady-state growth will be improbable). This asymmetry in modelling leads economists to favour labour-over-capital augmentation (Sheng 2017a, Sheng, L., Yin, Zhang and Wu 2022).

Based on this focus, some studies confirm that growth can be analysed solely in terms of labour augmentation. Researchers conclusively demonstrated that biased usage is a natural result of aggregate production under constant returns to scale (CRS) and capital-output growth equivalence ($g_K = g_Y$) (Johns & Scrimgeour 2008; Schlicht 2006). Others obtained similar findings after including factors such as human capital and intermediate products, and from the endogenous selection of labour and capital-enhancing technological progress (Acemoglu 2009; Irmen 2018; Nogueira 2022). They suggested that while both forms of factor augmentation are possible, only labour augmentation survives over the long term, while capital augmentation diminishes as the transition progresses toward a steady state. Many growth studies, notably that of Uzawa (1961), used a generic version of PF, or at the very least a constant elasticity of substitution (CES) PF, instead of the Cobb–Douglas PF. However, to obtain Harrod neutrality, the CRS or unitary CES between capital and labour is typically used along with additional de-generalised assumptions (such as equal growth among macro variables). Hence, the key conclusions of such research appear to be a byproduct of the underlying theories (Irmen 2018).

In practice, there is no specific reason for this influence of technological change on labour augmentation. The Cobb–Douglas PF has a unitary CES, but extensive data imply an elasticity considerably below one, which may be why its application in growth research is uncommon (Acemoglu 2003). Other researchers argue that a more generic PF with CRS can yield an accurate representation of reality. Adjusted for quality, it is evident that since 1947 the relative price of capital equipment in the US has been steadily falling (Gordon 1990). This can jeopardise the BGP and thus invalidates the claim that technological change is labour-augmenting, even under CRS, as the price drop implies $g_Y \neq g_K$ (Grossman et al. 2017). In addition, CRS may not apply to all nations or all time periods (Romer 1992), therefore these growth dilemmas suggest that capital-augmenting technological change should not be overlooked.

This study was intended to be generic in terms of non-CRS technology use and the imbalanced route due to $g_Y \neq g_K$, but as in previous studies, the authors made certain assumptions that can be restrictive. However, it was assumed that the PF is in Cobb–Douglas form to ensure that the analysis remains simple and straightforward. This assumption implies that the type of technological change has no bearing on factor augmentation in a steady state. All forms of factor augmentation are then possible, enabling to conduct a comparative research. The authors could thus extract more detailed conclusions with more intuitions under all forms of returns to scale, as the unitary elasticity of scale assumed in other research (i.e. CRS) is relaxed in this study. The authors did not consider any per-capita effective variables that only apply to labour and not to capital, which led other researchers to favour labour-over-capital augmentation. The incisive although difficult theoretical work of Grossman et al. (2017) provided more insights than other studies by combining both capital-augmenting technological development and human capital (Lucas 1988). To make this analysis as simple as possible, the study did not follow this direction, and the impact of human capital remained outside of the scope of the research. The aim was to examine the effects of dropping investment good prices, variable returns to scale, input-output elasticities, and labour availability on steady-state growth in output, capital, and per-capita income that is no longer balanced. Furthermore the authors show that while the labour and capital-augmentation technical developments are not immediately measurable, discriminating between them is unnecessary. As there were no data on their effects on growth, it is more theoretically useful to apply the Solow residual (also known as total factor productivity, or TFP) to obtain an indirect estimate.

The remainder of this paper is organized as follows. Section 2 describes a problem that arises from the use of effective elements of production (namely for effective capital but not for effective labour). Section 3 focuses on the reasons for technology-induced augmentation. Section 4 compares several factor-augmentation technologies. In Section 5, the authors argue that the Solow residual can be a useful growth accounting tool. Section 6 concludes the paper.

2. A puzzle arising from using effective factors of production

Lemma 1: $g_x = dx_t/(x_t d_t)$ denotes growth in any variable x_t at time t. The growth rates of variables (w_t, x_t, y_t, z_t) are related as $g_w = \alpha g_x + \beta g_y - \gamma g_z$ under $w_t = c{x_t}^{\alpha}{y_t}^{\beta}/{z_t}^{\gamma}$ with $(c, \alpha, \beta, \gamma)$ as constants.

Lemma 2: For the variables in the expression $z_t = \sum_{1\le i\le n} x_{it}$ at time t, namely, z_t, x_{it}; $1 \le i \le n$, their growth rates stick on the relationship as $g_z = \sum_{1\le i\le n} s_{it} g_{xi}$, note that $s_{it} = x_{it}/z_t$, and $\sum_{1\le i\le n} s_{it} = 1$. The two lemmas thus imply that growth rates may not be time-invariant unless steady-state growth is assumed.

Capital per effective worker at time t is defined as $k_{et} = K_t/(T_t L_t)$, where K_t is capital stock, L_t – labour supply, and $T_t (\ge 1)$ is the technology state, which presents the function of technical change in terms of labour enhancement as suggested in the literature. Similarly, output per effective worker is defined as $y_{et} = Y_t/(T_t L_t)$, where Y_t is total output (Sheng, Li and Wang 2017). Applying these two definitions (k_{et}, y_{et}) leads to two concerns. First, students may have ongoing difficulties in removing any ambiguity when initially studying macroeconomics. Second, the asymmetry in validity between capital and labour when using effective variables naturally leads to problems in advanced research into steady-state growth.

The first issue at the beginning of every macro course instruction is as follows. The labour-augmenting Solow model, with technological change at a constant rate of $g_T = \Delta T_t/T_t = \theta (> 0)$ is presented as

$$\Delta k_{et} = s y_{et} - (n + d + \theta) k_{et},$$

where *s* denotes the saving rate, *d* denotes capital depreciation, and $g_L = \Delta L_t/L_t = n$ is labour force growth. Students are taught that output per *effective* worker is constant in a steady state (i.e. $\Delta k_{et} = 0$) by definition, not per *actual* worker $y_t = Y_t/L_t$ (or income per person). The constant level is determined via a straightforward derivation as

$$y_e^* = [s/(n + d + \theta)]^{\alpha/(1-\alpha)} \tag{1}$$

if production is conducted using the formula $Y_t = {K_t}^{\alpha}{L_t}^{1-\alpha}$ with constant returns to scale (CRS). Under the assumption for the output elasticity of capital input, this indicates that output per effective worker ($y_e^* \downarrow$) falls with technical development ($\theta \uparrow$), as in Equation (1). Many students find it puzzling that technological advancement (which is an extremely positive phenomenon) leads to a decrease in production per effective worker (a seemingly negative effect). They may remain sceptical even after being assured that income per capita y_t should be given special attention and that y_{et} is only an auxiliary variable. Some students continue to doubt whether income per capita $y_t^* = y_e^*(\theta)T_t$ can fall with greater technology if T_t or θ is assessed incorrectly. The difficulty is that the influence of technological progress cannot be directly quantified, as statistics for T_t and θ are not available.

The second challenge mentioned above has yet to be resolved in growth research. The use of the effective units T_tL_t of the labour supply underlies the labour-augmentation Solow model. The constant output per effective worker in the steady state, $y_e^* = Y_t/(T_tL_t)$, implies

$$g_Y = g_T + g_L.$$

As shown by Lemma 1, output Y_t rises at a rate of $n+\theta$, whereas income per capita $y_t = g_Y - g_L$ grows at θ. This and other findings were obtained using CRS technology and the *capital per effective worker* $k_{et} = K_t/(T_tL_t)$. However, when using *effective capital per worker* $k_{et}{}^T = T_tK_t/L_t$, a comparable intuitive result cannot be produced. In this example, correctly defining $y_{et}{}^T$ as a function of $k_{et}{}^T$ is not apparent. There is a constant level of $k_e^{*T} = T_tK_t/L_t$ in the steady state if one utilises $k_{et}{}^T$ together with $y_t = Y_t/L_t$ to obtain the growth findings, so that

$$g_T + g_K = g_L \tag{2}$$

$g_k = g_K - g_L = -\theta < 0$ follows from Lemma 1. This finding is clearly undesirable, as it implies that effective capital per worker is not an adequate theoretical auxiliary variable for rational analysis. The shift away from capital-enhancing technology transformation in growth research may be due to this. The question of whether one should completely forgo capital augmentation in favour of only labour augmentation remains.

3. Technology induced augmentation for more than one factor

The explanation above demonstrates that the idea of effective factors applies to labour but not to capital. Many academics are now focusing on the effects of labour-augmenting technology change and the steady-state growth theorem because of this validity imbalance. Capital K_t / investment I_t, consumption C_t, and output Y_t must rise at constant and equal rates in a steady state, according to this extremely restricted theory, and production technology should be labour-augmenting to achieve such a balanced growth path (BGP). Some economists argued for Harrod neutrality by demonstrating that the influence of technology as a scalar on labour is dependent on two factors: CRS and $g_Y = g_K$ (Schlicht 2006; Acemoglo 2009; Sheng 2017a; Sheng *et al*., 2023). However, putting aside the CRS, the question of whether $g_Y = g_K$ holds in practice for all nations in the long term remains unanswered.

Unfortunately, if any of the following conditions are not met, the assumption of $g_Y = g_K$ may not hold: (i) the saving rate is constant, (ii) the trade account is balanced, (iii) the capital price is stable relative to final output, or (iv) growth rates are equal between any two variables (Y_t, C_t, I_t, NX_t), where NX_t is net trade. Such conditions may be rigorously demonstrated through straightforward methods, or can be validated from the facts. Saving rates have dropped in the US and other OECD countries but have risen in China and other emerging-market economies over lengthy periods of time, according to real-world statistics (Yin and Sheng 2021; Sheng, Yin and Zhang 2022). Trade is never generally regarded as balanced in any country, but $NX_t \neq 0$, and global imbalances are frequently the basis of trade wars and international disagreements (Obstfeld and Rogoff 2009). The evidence demonstrates that when adjusted for quality, relative price p_t of investment goods I_t has been declining dramatically in the USA since 1947 (Gordon 1990), indicating that capital-augmenting technical progress is inherent in each new generation of capital equipment (Grossman et al. 2017). In addition, whereas consumption

C_t seems to move along with production Y_t (i.e. $g_C \approx g_Y$), investment I_t is significantly more volatile, in addition to the volatility in current-account balance NX_t.

Next it was examined how a falling p_t may invalidate the $g_Y = g_K$ assumption. The cost of new investment is used to price capital goods in terms of final products, with $p_t = 1/q_t$ and $\sigma \equiv g_q = -g_p > 0$ as the price falls, in which one unit of output is translated to q_t units of capital and g_q represents investment-specific technological change. Under balanced trade, an economy's resource restriction should then be expressed as

$$Y_t = C_t + I_t/q_t \quad \text{with} \quad g_Y = g_K - g_q \neq g_K. \tag{3}$$

Using Lemmas 1 and 2, which have already been established in the literature, such steady-state but imbalanced growth ($g_Y \neq g_K$) can be demonstrated (Grossman et al. 2017). This conclusion refutes the argument that the production function (PF) can only tolerate Harrod neutrality or labour-augmenting technological change if $g_Y = g_K$ is assumed. Thus, Solow-neutral (capital-augmenting) or Hicks-neutral (scaling-effect) technical advancement cannot be ruled out, particularly when the PF does not satisfy the CRS condition. The question is then whether Uzawa's steady-state growth, or BGP, can be maintained after the PF is no longer limited to labour augmentation.

4. A comparison of different factor-augmenting technologies

Growth paths may be unbalanced under the realisation of steady-state growth when comparing different types of factor augmentations. Effective factors in per capita terms will not entirely prevent inaccurate outcomes as a result of Equation (2) because they are auxiliary (i.e. not key) variables. The Cobb-Douglas PF was used to ensure that growth analysis remains tractable, as in the classic literature (Solow 1956), with $0 < \alpha < 1$ and $0 < \beta < 1$ representing the output elasticity of K_t and L_t, respectively. The CRS assumption is modified to generalise this study and its outcomes, with $\tau = \alpha + \beta - 1$ (>, =, or < *0*) used to indicate variable returns to scale (VRS). Clearly, three different technologies leading to growing, constant, or declining returns to scale correspond to three alternative situations of τ >, =, or correspondingly < 0.

The three main growth analysis building blocks were: (i) the PF under VRS with no technical change, $Y_t = {K_t}^{\alpha} {L_t}^{\beta}$ (denoted as PF_O), (ii) the national income account (NIA) under no trade, $Y_t = C_t + p_t I_t$ (as in Equation (3)), and (iii) the capital accumulation process (CAP), $\Delta K_t = I_t - dK_t$. An in-depth examination of the Solow model revealed many types of technological change, which then led to PF becoming $Y_t = {K_t}^{\alpha}(T_t L_t)^{\beta}$ (denoted as PFL) under Harrod neutrality, $Y_t = (T_t K_t)^{\alpha} {L_t}^{\beta}$ (denoted as PFK) under Solow neutrality, and $Y_t = A_t {K_t}^{\alpha} {L_t}^{\beta}$ (denoted as PFA) under Hicks neutrality. Here, the study abused the notation for expositional convenience; CAP was recast as $g_K + d = I_t/K_t$ such that $g_I = g_K$ as usual under the assumption of a steady state from Lemma 1. Based on Lemmas 1 and 2, NIA was used to establish that $g_Y = g_C = g_I + g_p \neq g_K$ for steady-state (but not balanced) growth. As $g_s = (g_Y - g_C)C_t/S_t = 0$, the saving rate s_t must drop to a constant *s* when $g_Y = g_C$.

According to CAP, NIA, and the three PFs (namely PF_L, PF_K, PF_A), the Solow model can be used to calculate the equilibrium levels and growth rates of important macro variables. The answers to the model's three scenarios are given below.

(I) In a steady state under Harrod neutrality with PF_L,

$$y_t^* = \left\{ \frac{s\, q_t}{g_K + d} \left(T_t^{\beta} L_t^{\tau} \right)^{\frac{1}{\alpha}} \right\}^{\frac{\alpha}{1-\alpha}}, \tag{4}$$

where $g_Y = \frac{1}{1-\alpha}[(n+\theta)\beta + \alpha\sigma]$, $\quad g_y = \frac{1}{1-\alpha}(\alpha\sigma + \beta\theta + \tau n)$,

and $g_K = \frac{1}{1-\alpha}[(n+\theta)\beta + \sigma]$, $\quad g_k = \frac{1}{1-\alpha}(\sigma + \beta\theta + \tau n)$.

Clearly, capital $sq_t/(g_K + d)$, labour ${L_t}^{\tau/\alpha}$, and technological state ${T_t}^{\beta/\alpha}$ exist along with input-output elasticities (α, β) and VRS (τ >, =, or < 0). All influence the equilibrium level y_t^* of per-capita income.

Five observations can be derived from the results in Equation (4). First, as $g_Y \neq g_K$ and $g_y \neq g_k$, production and capital grow at distinct rates, whether in aggregate or per capita. Second, a faster decline in capital price p_t is beneficial to production and capital growth as the reduction results in a greater σ. Third, VRS technologies have different effects on production and capital growth rates in per capita terms. If production has growing (decreasing) returns to scale under $\tau > (<) 0$, per capita output and capital expand faster (slower). CRS do not have this effect. Fourth, if one sets $\tau = 0$ (i.e. $\alpha + \beta = 1$), the model is reduced to the case of CRS, but $g_Y \neq g_K$ is preserved. In this situation,

$$g_Y = \frac{\alpha\sigma}{1-\alpha} + \theta + n, \quad g_y = \frac{\alpha\sigma}{1-\alpha} + \theta, \quad g_K = \frac{\sigma}{1-\alpha} + \theta + n, \quad g_k = \frac{\sigma}{1-\alpha} + \theta.$$

The price of investment products thus appears to have a greater influence on capital growth than output growth. Fifth, one can further decrease both y_t^* in Equation (4) to $T_t y_e^*$ in Equation (1), and growth rates to $g_Y = g_K = n + \theta$ and $g_y = g_k = \theta$ by setting $p_t = 1$ (i.e. $q_t = 1$ and $\sigma = 0$) along with $\tau \equiv 0$ (i.e. $\alpha + \beta = 1$). The Uzawa theorem, or BGP, is now shown to be a specific example of the model. The generic model's Equation (4) conclusions are more useful than those for a particular case. In this model, capital output elasticity is directly related to growth, but in the Uzawa-style models, it is not. A novel finding in this model was that a high level of such elasticity promotes capital and output growth.

(II) In a steady state under Solow neutrality with PF_K,

$$y_t^* = \left\{\frac{sq_t}{g_K+d} T_t L_t^{\frac{\tau}{\alpha}}\right\}^{\frac{\alpha}{1-\alpha}}, \tag{5}$$

where $g_Y = \frac{1}{1-\alpha}[\alpha(\sigma + \theta) + \beta n], \quad g_y = \frac{1}{1-\alpha}[\alpha(\sigma + \theta) + \tau n],$

and $g_K = \frac{1}{1-\alpha}(\sigma + \alpha\theta + \beta n), \quad g_k = \frac{1}{1-\alpha}(\sigma + \alpha\theta + \tau n).$

Equation (5) provides four observations. First, as $g_Y \neq g_K$ and $g_y \neq g_k$ show, production and capital rise at different rates. Second, a greater decline in capital price p_t is favourable to further macroeconomic development, as it suggests a higher σ. Third, if the technology follows rising (decreasing) returns to scale under $\tau > (<) 0$, production per capita and capital per capita rise faster (slower). CRS have no such impact. Fourth, the output elasticity α of capital has a favourable impact on the growth rates of all of the macro variables.

(III) In a steady state under Hicks neutrality with PF_A, with g_A also denoted by θ,

$$y_t^* = \left\{\frac{sq_t}{g_K+d} (A_t L_t^{\tau})^{\frac{1}{\alpha}}\right\}^{\frac{\alpha}{1-\alpha}}, \tag{6}$$

where $g_Y = \frac{1}{1-\alpha}(\theta + \beta n + \alpha\sigma), \quad g_y = \frac{1}{1-\alpha}(\alpha\sigma + \theta + \tau n),$

and $g_K = \frac{1}{1-\alpha}(\theta + \beta n + \sigma), \quad g_k = \frac{1}{1-\alpha}(\sigma + \theta + \tau n).$

From Equation (6), one can make observations similar to those mentioned above. First, the $g_Y \neq g_K$ and $g_y \neq g_k$ growth rates of production and capital are different. Second, a greater decline in capital price pt accelerates their expansion by implying a higher σ. Third, if production displays growing (decreasing) returns to scale under $\tau > (<) 0$, then output and capital expand faster (slower) in per capita terms. CRS have no such impact. Fourth, the capital output elasticity has a favourable impact on growth rates.

Three findings emerged from comparing the above three situations of technical change (I, II, III) stated in Equations (4), (5), and (6). First, in each of the three scenarios, technical progress, T_t or A_t, has a distinct influence on production growth. The coefficient on θ in g_Y and g_y is 1 in case I, $\alpha/(1-\alpha)$ in case II, and $1/(1-\alpha)$ in case III; this also holds true for capital growth. Second, in all three scenarios investment price p_t has the same influence on production growth, which is also true for capital growth, but to a greater extent. For all cases (I, II, and III), the coefficient on σ is $\alpha/(1-\alpha)$ in (g_Y, g_y) and $1/(1-\alpha)$ in (g_K, g_k). Third, if the technology exhibits growing returns to scale (i.e. $\tau > 0$), labour force L_t directly adds to production per capita y_t^* in a steady state but reduces such output if declining returns to scale prevail (i.e. $\tau < 0$). In the case of CRS (i.e. $\tau = 0$), there are no such impacts.

The main implications derived from the above comparative analysis of factor augmentations can be summarised as follows.

Proposition 1: Six of the findings are resistant to various types of factor augmentation. (i) Capital and output both grow at different rates. (ii) As investment prices decline, capital and output growth accelerates. (iii) With increasing (decreasing) returns to scale, output and capital per capita rise faster (slower). The authors did not find this effect with CRS. (iv) The investment price has a greater influence on capital growth than on output growth. (v) A high input-output elasticity is beneficial to capital and output growth. (vi) Under increasing returns to scale, the labour force contributes to per-capita income, but under declining returns to scale it reduces per-capita income. The study did not find this effect with CRS.

5. The Solow residual as a more effective tool for growth accounting

Favouring a labour or a capital-augmentation formulation of PF makes no sense, as the influences of technology on output are prevalent in all areas of the production process. Assume that in a general situation, such influences are represented by effective capital $\varphi_t K_t$ and effective labour $\psi_t L_t$ for $\varphi_t > 1$ and $\psi_t > 1$. T_t, which was misused in the previous discussion, is then substituted by φ_t for capital augmenting and ψ_t for labour augmenting in this example. Given the initial PF$_o$: $Y_t = K_t{}^{\alpha} L_t{}^{\beta}$, these impacts can feasibly be rewritten as PF$_{LK}$: $Y_t = (\varphi_t K_t)^{\alpha}(\psi_t L_t)^{\beta}$, which may be flexibly rewritten as a Harrod-neutral PF$_L$: $Y_t = K_t{}^{\alpha}(T_L L_t)^{\beta}$ with $T_L = \varphi_t{}^{\alpha/\beta}\psi_t (> \psi_t)$, or a Solow-neutral PF$_K$: $Y_t = (T_L K_t)^{\alpha} L_t{}^{\beta}$ with $T_L = \psi_t{}^{\beta/\alpha}\varphi_t (> \varphi_t)$. Thus, unsurprisingly, some economists continue to focus on capital-augmenting growth despite significant labour-augmenting technological implementation. PF$_{LK}$ can be further rewritten as a Hicks-neutral (i.e. scale-augmenting) PF$_A$: $Y_t = A_t K_t{}^{\alpha} L_t{}^{\beta}$ with $A_t = \varphi_t{}^{\alpha}\psi_t{}^{\beta} (> 1)$. Hence, technological effects simply augment the scale of output, with the two effectiveness-related multipliers (φ_t, ψ_t) merging into a new scale multiplier A_t (larger than either $\varphi_t{}^{\alpha}$ or $\psi_t{}^{\beta}$). Therefore, any technological change is simply reflected in its produced scale effect. This impact has little to do with VRS types, or the CRS that the BGP use to maintain Harrod neutrality, and thus it is unnecessary to theoretically distinguish between labour and capital augmentations.

From an empirical standpoint, favouring one type of augmentation over another does not appear to make sense, as directly or explicitly measuring technological advancement and its effects on production is challenging. No official statistics or other data are available, hence the influences of sources of output growth are commonly assessed as a Solow *residual* (denoted as δ). As many types of factor augmentations can be converted to a scale effect, deriving the residual from the PFA is sufficient. Consider how the Solow growth accounting formula (SGAF) was derived from PF$_A$; when Lemma 1 is applied to PF$_A$, the result is $g_Y = \alpha g_K + \beta g_L + g_A$, which is translated as SGAF: $\delta (\equiv g_A) = g_Y - (\alpha g_K + \beta g_L)$. Note that the Solow residual δ is proportional to $\beta\theta$ in case I, to $\alpha\theta$ in case II, and to θ in case III. Under PF$_{LK}$, which is equal to $\delta = \alpha g_{\varphi} + \beta g_{\psi}$, this residual may become more complicated. However, there is no reliable method for estimating θ directly, or either g_{φ} or g_{ψ}. Under CRS, $F_K = r$, $F_L = w$, and $Y = F_K K + F_L L$ are the results of profit maximisation (for factor pricing) and the Euler theorem (for income distribution). When these are added together, the income shares of investors and workers in aggregate production are $\alpha = rK/Y = s_K$ and $\beta = wL/Y = s_L$, respectively. Thus, the SGAF assumes the form of

$$\delta = g_Y - (s_K g_K + s_K g_L) \tag{7}$$

when CRS is followed by production. Due to the availability of official data for all of the variables on the right-hand side of Equation (7), impact δ of technical development given on the left side is only implicitly observable in the form of the calculated *residual as a scale effect* by indirect treatment (7). Clearly, intentionally distinguishing between labour and capital augmentations is not experimentally beneficial.

The empirical research has long had its own set of challenges. In many countries, the Solow residual δ in Equation (7), often known as total factor productivity (TFP), is used to empirically analyse technical advancement as a driver of economic growth. The TFP encompasses more than technical change and is a proxy for overall efficiency or presumed ignorance of the growth process, as it is actually a combination of measurement errors in the available data and inevitable omissions of other factors from growth models, in addition to technological advancements and efficiency gains. These elements are difficult to quantify, and no relevant data are available. If CRS do not apply to a given economy, $rK + wL = (\alpha + \beta)Y$ may not precisely match its Y, resulting in $s_K + s_L(= 1 + \tau) \neq 1$, affecting the precision of predicting its TFP using Equation (7). When real production does not have a unitary elasticity of substitution between K and L or a unitary elasticity of scale, continuing to focus on labour-augmenting technology change may not increase estimation accuracy.

6. Concluding remarks

The authors found that from an empirical perspective, distinguishing between labour and capital augmentation is unnecessary. Any type of technical change is simply reflected in the scale effect that it induces, as different factor augmentations merge to produce a scale effect, regardless of VRS types. Favouring one augmentation over another does not make sense, as it is difficult if not impossible to directly measure the specific effects on capital or output from each type of technological change using the available data, therefore all of these effects should be jointly measured as a Solow residual via indirect computation.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.

Gordon, R. J. (1990). *The Measurement of Durable Goods Prices*. University of Chicago Press.

Grossman, G. M., Helpman, E., Oberfield, E., & Sampson, T. (2017). Balanced growth despite Uzawa. *American Economic Review*, 107(4), 1293-1312.

Irmen, A. (2018). A generalized steady-state growth theorem. *Macroeconomic Dynamics*, 22, 779-804.

Lucas, R. E. Jr. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1), 3-42.

Nogueira, A. S. (2022). Management of natural resources in protected areas: Interinstitutional dialogue, social capital, and agency in the transition to agroecological systems. *Tempo Social*, 34(3), 341-373.

Obstfeld, M., & Rogoff, K. (2009). Global imbalances and the financial crisis: Products of common causes. *Working Paper*, UC Berkeley and Harvard University.

Romer, P. M. (1992). Growth based on increasing returns due to specialization. *American Economic Review*, 77(2), 56-621.

Schlicht, E. (2006). A variant of Uzawa's theorem. *Economics Bulletin*, 5(6), 1-5.

Sheng, L. (2017a). Factor determining the success or failure of a tourism tax: A theoretical model. *Tourism Review,* 72(3), 274-287.

Sheng, L. (2017b). Explaining urban economic governance: The city of Macao. *Cities*, 61(2017)、 96-108.

Sheng, L., Li. T., & Wang, J. (2017). Tourism and externalities in an urban context: Theoretical model and empirical evidence. *Cities*, 70(2017), 40-45.

Sheng, L., Yin, Y. C., & Zhang, A. N. (2022). Theorizing on tourist flows and business cycles in casino cities. *Argumenta Oeconomica*, 48(1), 187-199.

Sheng, L., Yin, Y. C., Zhang, A. N., & Wu, J. W. (2022). Modelling economic structure: The perspective of tourist cities. *Argumenta Oeconomica*, 48(2), 135-150.

Sheng, L., Yin, Y., Zhang, A., Wu, J., & Yang, Z. (2023). Modelling casino hospitality business cycles. *Argumenta Oeconomica*, 1(50), 33-42.

Sheng, L., & Zhang, Z. (2022). Revisiting global imbalances: A comparative analysis of income and consumption inequality. *Economics & Politics*, 34(2), 382-390.

Solow, R. M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70, 65-94.

Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. *Review of Economic Studies*, 28, 117-24.

Yin, Y., & Sheng, L. (2021). Theorizing about global imbalances: An inequality perspective. *Argumenta Oeconomica*, 46(1), 169-182.

Received: June 2022