Argumenta Oeconomica
Year 2024, Vol. 52, No. 1
ISSN 2720-5088

An epidemic and economic growth. A medium-term perspective

Monika Bolińska
Faculty of Management and Social Communication, Jagiellonian University in Kraków, Poland
e-mail: monika.bolinska@uj.edu.pl
ORCID: 0000-0003-0361-7148

Paweł Dykas
Faculty of Management and Social Communication, Jagiellonian University in Kraków, Poland
e-mail: pawel.dykas@uj.edu.pl
ORCID: 0000-0002-3317-941X

Tomasz Tokarski
Faculty of Management and Social Communication, Jagiellonian University in Kraków, Poland
e-mail: tomasz.tokarski@uj.edu.pl
ORCID: 0000-0002-9551-0892

Rafał Wisła
Faculty of Management and Social Communication, Jagiellonian University in Kraków, Poland
e-mail: rafal.wisla@uj.edu.pl
ORCID: 0000-0002-7895-6243

Quote as: Bolińska, M., Dykas, P., Tokarski, T., & Wisła, R. (2024). An epidemic and economic growth. A medium-term perspective. *Argumenta Oeconomica*, 1(52), 18-36.

DOI: 10.15611/aoe.2024.1.02

JEL Classification: E1, H8, I1

Abstract: The described model of economic growth under conditions of an epidemic does not directly refer to the social and economic situation in the years 2020 and 2021, i.e. to the COVID-19 pandemic. It is aimed to identify the consequences of any epidemic leading to severe social losses (high percentages of the infected and dead, limited interpersonal contacts as a result of lockdown, a lowered level of general individual and social well-being) and economic losses (a fall in production as a result of the collapse of aggregate demand and a reduction in the supply capacity of the economy, and consequently a decreased rate of capital accumulation).

The described epidemiological-economic model combines the SIR epidemiological model from 1927 (with its modification proposed in 2020 by Bärwolff) and the neoclassical growth model proposed by

Solow in 1956. The authors expanded the concept by lockdown restrictions with various levels of their severity, considering specific rules for their implementation, and by guidelines for scenario analyses of changes in the value of social and economic activity index, in the value of aggregate production, in the capital stock accumulated in the economy and in social utility. The effect of an epidemic on medium-term economic growth was also considered, for various levels of economic development (measured by the distance of input capital stock from that formed in a long-term equilibrium in the Solow model).

The principal conclusions drawn from the discussion are listed below. First, the introduction of the same restrictions imposed on social and economic activity throughout the territory of a country poses a considerable risk of error in assuming a uniform spread of the epidemic. Second, a rapid introduction of lockdown measures has a stronger effect on accumulated social and economic activity than a continual process of gradual imposing and lifting restrictions on that activity. Third, a scenario of a rapidly imposed severe lockdown has the most drastic consequences for the economy, causing both the sharpest drop in production, the longest time of remaining on the path of relatively lowest growth, and the most dramatic expected accumulated decrease in production. Similar conclusions are drawn as to the changes in social utility.

Keywords: SIR model, Solow model, epidemic, lockdown, social utility

1. Introduction

This study aimed to assess the effect of an epidemic on medium-term economic growth (i.e. over five years). The analysis was conducted using an epidemiological-economic model that combines the SIR (Susceptible-Infectious-Recovered) epidemiological model proposed by Kernack and McKendrick (1927) with a neoclassical model of economic growth proposed by Solow (1956).

The original SIR model does not consider restrictions imposed on social and economic activity in response to the spread of an epidemic. An analysis of the process of the spread and subsidence of an epidemic was made using the SIR model as modified by Bärwolff (2020), who assumed that governments impose restrictions on social and economic life when an epidemic begins to spread out of control (the percentage of infected people exceeds a certain critical level defined in an arbitrary manner by the government). Bärwolff also suggested that the more restrictive lockdown is introduced, the slower the pace of epidemic spread.

Bärwolff's study was based on the assumption that the state introduces lockdown measures rapidly in an arbitrary manner (within a period or at defined time intervals). In this epidemiological-economic model, the authors assume that the level of lockdown severity is defined using a specific functional rule. Namely, it was assumed that the severity index of a lockdown (also known as the index of social and economic activity) is an analytical function of the percentage of the infected. If the percentage grows, the government follows the rule described by the function (and does not use arbitrary criteria), imposing restrictions on social and economic life.

In the economic module of the model (as in the model proposed by Sahbani et al., 2019), it is expected that the value of production in the economy depends on the capital stock available in the economy and the percentage of people that are not infected. However, the authors also assumed (unlike Sahbani et al., 2019) that the value of production is also affected by the severity of lockdown. This can be substantiated by the fact that a reduced level of social and economic activity affects both the supply and demand sides of the economy. With the spread of an epidemic, the infected population of workers is (naturally) excluded from participation in production processes and this limits the supply potential of the economy. Additionally, the more severe a lockdown , the larger the segment of the economy completely or partly prevented from normal operations (e.g. the catering, hospitality or transport sectors). In consequence, the rate of unemployment rises and/or incomes in the population fall. These processes result in a fall in aggregate demand in the economy and a Keynesian reduction in the value of production combined with an increased waste of production capacity available in the economy.

Moreover (as in the growth model proposed by Solow), the authors understand an increase in capital stock as the difference between investment in the economy (representing a constant proportion in the value of production) and capital depreciation. Thus, if the percentage of the infected rises, the falling value of production causes a drop in investment, and this reduces the growth rate of capital stock. The reduced rate of capital accumulation limits the supply capacity of the economy and this entails adverse short, medium and long-term consequences.

In the proposed epidemiological-economic model, the spreading epidemic and the accompanying lockdown affect the production processes. If the percentage of the infections rises, the value of production falls, whereas restrictions imposed on social and economic life result in a reduction in the spread rate of the epidemic, reducing the percentage of the infected. This has a favourable effect on both the supply and demand side of the economy.

The model does not consider the direct impact of the economy on the course of the epidemic. This can be justified by the fact that the macroeconomic module generates only such aspects as production, capital resource, investments or consumption. However, it seems that these variables do not directly affect the speed at which an epidemic spreads.

The epidemiological-economic model proposed in this study has no explicit analytical solution, hence the study also calibrated its parameters and solved it numerically (at calibrated values of those parameters). The results of numerical simulations covering a period of five years under conditions of very severe and mild lockdown restrictions were also provided. The effect of restrictions imposed on social and economic life on economic growth was studied both in the case in which the state imposes them following a functional rule, and for restrictions imposed rapidly at an arbitrary level.

2. Review of the literature

The SIR epidemiological model and its subsequent augmentations have been used principally in the mathematical modelling of contagious disease spread since the publication of the pioneering study by Kermack and McKendrick (1927) (see: Murray, 2003; Ruan, 2007; Xiao, Ruan 2007; Fei-Ying, Wan-Tong, Zhi-Cheng, 2015; Jardón-Kojakhmetov, Kuehn, Pugliese, Sensi, 2021). An interesting trend in this field of research is represented by the studies into contagious disease spread considering various patterns of spatial mobility of the population (see e.g. Sattenspiel and Dietz, 1995; Arino and van den Driessche, 2003) and of animals carrying viruses that are dangerous to people (Ruan and Wu, 2009).

The problem of distorted economic growth caused by a dramatic increase in incidence and other health issues was not systematically studied in economics over recent decades. However, it must be emphasised that in the 1980s and 1990s the economic effects of HIV/AIDS in Asia (Bloom and Lyons, 1993) and in selected countries of Europe, Africa, North America and South America (Bloom and Mahal, 1995; Kambou et al., 1992) were analysed. For example, Bloom and Mahal (1995, 1997) argued that the HIV/AIDS epidemic had no material effect on the rate of growth of income per capita in 51 developed and industrialised countries in the period 1980-1992. Two decades later, Cuesta (2010) came to a similar conclusion about Honduras, the country most severely affected by the HIV/AIDS epidemic in South America.

The current scale and rate of spread of the COVID-19 pandemic caused by the coronavirus entails serious disturbances in social and economic life. The 2020+ pandemic has been not only the most serious global health crisis since the Spanish flu of 1918, but also one of the most economically expensive pandemics on a global scale. In response to the chain of events observed, several measures are being currently considered. Alvarez et al. (2020) and Atkeson (2020) addressed the problem of optimisation of the severity level of a lockdown. They used the SIR model under conditions of changing economic activity of the population and enterprises. This research trend is also represented by Bärwolff (2020), whose study provided a starting point for this discussion. Bärwolff argued that a lockdown leads only to a displacement of the climax of the pandemic, but not actually to an efficient flattening of the curve of the number of infected people. He also pointed out the importance of social distancing. A similar conclusion was drawn by Lik Ng (2020), who indicated the adverse effects of a lockdown policy treated as the principal method preventing the spread of pandemic. Coccia (2021) highlighted that the longer period of lockdown has a negative impact on economic growth. The effects of the macroeconomic lockdown and its transmission to the rest of the economy differ by company size and across sectors. Using the Spanish context for micro, small, medium, and large companies, Pedauga et al. (2021) distinguished the direct and indirect effects caused by the COVID-19 pandemic.

Research into trade-off in public choices was also initiated in 2020. Aum et al. (2020) analysed the trade-off between GDP and public health under pandemic conditions, arguing that a lockdown not only limits the spread of pandemic, but also mitigates the accumulated GDP loss in the long run. If no lockdown measures are taken during a pandemic, mass quarantining is necessary, leading to adverse economic effects. The self-employed who achieve relatively low income form the group exposed to the most severe consequences of a lockdown. Brock and Xepapadeas (2020) adopted an even wider perspective, suggesting that continuous growth of consumption activities, capital accumulation and climate change could increase the exposure of society to the risk of infection. In their opinion, a policy preventing the spread of epidemic should consist of two components: the first includes short-term measures, while the second – economic policies aimed at changing consumption patterns and addressing climate change. For example, the spillover effect of China's economy on economic growth is far greater regarding energy consumption (Wang and Zhang, 2021).

The Solow growth model (1956) is also used to assess the current and forecast consequences of the epidemic. Cuddington (1993) applied it to analyse the growth path of per-capita GDP in the context of HIV/AIDS epidemics and its demographic consequences. The Solow model used by him indicated a material risk of reduction in the GDP growth rate in Tanzania by the year 2010. Cuddington and Hancock (1994), adopted the same methodological approach to assess the effect of HIV/AIDS on the economy of Malawi. Delfino and Simmons (2005) identified significant empirical links between the health structure of the population and the productive system of an economy that is subject to infectious disease, in particular tuberculosis. Infectious diseases affect the size of the labour force and the productive capacity of the economy. Their study combined a Lotka-Volterra type system (capturing the dynamics of TB epidemics) with a Solow growth model where output is produced from capital and healthy workforce. The demographic-epidemiological parameters of the Lotka-Volterra type system are functions of GDP per healthy worker.

McDonald and Roberts (2006) used standard neoclassical growth models in their analyses of the macroeconomic impact of the HIV/AIDS epidemic. They employed the aggregate production function as a Cobb-Douglas function with labour and three types of capital: physical, education and health (as significant components of human capital), highlighting the importance of a research agenda that will provide a greater understanding of the mechanisms by which sustained epidemics impact upon health capital and macroeconomic performance.

Lovasz and Schipp (2009) used the model proposed by Mankiw et al. (1992), augmenting the Solow growth model by the process and effects of the accumulation of human capital. Their study covered Sub-Saharan Africa, suggesting that the level of education and health capital and the rate of epidemic spread have a strong effect on aggregate macroeconomic indicators. The effect of the HIV virus is not the same in all countries, and even within individual countries. The economies characterised by developed healthcare infrastructures are capable of providing means aimed to prevent a rapid spread of an epidemic in its early phase. Additionally, Lovasz and Schipp (2009), when analysing the problem of the accumulation of human capital under epidemic conditions, stated that the loss of human capital due to an epidemic does not always entail the same consequences. The level of education and number of skilled workers and their outflow from manufacturing processes due to an epidemic affects the GDP growth rate to a varying extent. Similarly, the social capital stock is interrelated with economic growth under epidemic conditions.

Until recently, HIV and malaria were considered principal health issues with their disastrous effect on poor African countries. The 2020+ pandemic displays characteristics of a global disaster, which poses a hazard to the macroeconomic stability of most economies of the world.

The above review of the principal trends in research into the effect of epidemics on GDP growth leads to an epidemiological-economic model including functional and arbitrary lockdown rules that cause changes in the level of social and economic activity. The value of aggregate production is affected by the capital stocks, the rising percentage of infected people that reduces investment, the rate of capital accumulation, and the scale of lockdown restrictions.

The model is not strictly related or limited to the COVID-19 pandemic as it is useful in analysing the effects of any epidemic that leads to material social losses (a high percentage of infected and dead people, limited interpersonal contacts due to the implemented lockdown measures) and economic losses (a drop in production caused by a collapse of aggregate demand and a reduction in the supply capacity of the economy, and consequently in the rate of capital accumulation).

3. Methodology

3.1. The epidemiological module (a modified SIR model)

The epidemiological module of the described model incorporates the following assumptions about the epidemic spread and subsidence process[1]:

1) The population is divided into three segments: people who were not infected so far, those infected, and recovered and dead. The percentage of people who were not infected is denoted by S (*susceptible*), the percentage of infected people by *I* (*infectious*), and the percentage of recovered and dead[2] by *R* (*recovered*). Consequently, the following is true at each time $t \in [0, +\infty)$: $S(t) + I(t) + R(t) = 1 \land S(t),\ \ I(t),\ \ R(t) \in [0,1]$.
2) The increase in the percentage of susceptible people $\dot{S}$ is described by the differential equation

$$\dot{S}(t) =- \beta\kappa\big(I(t)\big)S(t)I(t), \tag{1}$$

where $\beta \in (0,1)$. The greater the parameter *β* is, the faster (*ceteris paribus*) the analysed epidemic spreads. Equation (1) implies that a drop $-\dot{S}$ in the percentage of the infectious (as in the original SIR model) is proportional to the percentages of susceptible people *S* and infected people *I*, and, as in the study by Bärwolff (2020), to the index $\kappa \in [0,1]$. The index describes the level of social and economic activity of the population. *κ*=1 denotes normal social and economic activity (carried out when there is no epidemic) while *κ*=0 represents circumstances under which that activity is completely locked. Equations (1) and (2) show that the lower value is assumed by the index *κ* of social and economic activity (i.e. the more restrictive the lockdown), the lower the rate of growth of the infectious group and (thus) the lower the rate of epidemic spread.

---

[1] It was implicitly assumed that all variables discussed below (both epidemiological and macroeconomic) represent differentiable functions of time $t \in [0, +\infty)$. The expression $\dot{x}(t) = \frac{dx}{dt}$ denotes a derivative of the variable *x* with respect to time *t*, i.e. a change in the value of variable x at time *t*.

[2] The percentage R can be decomposed into the percentage of recovered H and the percentage of dead D (where $H,\ D\ \in (0,1)$). Then: $\dot{R}(t) = \dot{H}(t) + \dot{D}(t)$, and by equation (3): $\dot{H}(t) + \dot{D}(t) = \gamma I(t)$. Assuming additionally that the recovered constitute the h ($h \in (0,1)$) part of the infected (the percentage of the infected who died equals $1 - h$), the following relations occur: $\dot{H}(t) = \gamma hI(t) \land \dot{D}(t) = \gamma(1 - h)I(t)$. In the numerical simulations of the accumulated percentage of the dead described below, it is assumed that h=0.97.

3) An increase in the percentage of infected $\dot{I}$ equals the difference between the current increase in the percentage of infected $\beta\kappa IS = -\dot{S}$ and the recorded percentage of recovered and dead γI, where $\gamma \in (0, \beta)$ denotes the proportion of recovered and dead in the group of the infected. Thus, the following relation occurs:

$$\dot{I}(t) = -\dot{S}(t) - \gamma I(t) = \beta\kappa\big(I(t)\big)S(t)I(t) - \gamma I(t), \tag{2}$$

4) A constant proportion γ in the infected group, recover or die. Hence[3]

$$\dot{R}(t) = \gamma I(t). \tag{3}$$

5) The index κ of social and economic activity during the epidemic (under non-arbitrary lockdown conditions) depends on the percentage of infected I. The relations between κ and I are described by the equation:

$$\kappa\big(I(t)\big) = 1 - I^{\sigma}(t), \tag{4}$$

where $\sigma > 0$.

If the parameter $\sigma \to +\infty$, the model described by equations (1)-(4) is consistent with the SIR model proposed by Kernack and McKendrick (because then at any time $t \in [0, +\infty)$ $\kappa = 1$). Assuming a finite value of parameter σ, the SIR model is modified by the Bärwolff augmentation.

Equations (1), (2) and (4) give the following system of differential equations[4]:

$$\begin{cases} \dot{S}(t) = -\beta\big(1 - I^{\sigma}(t)\big)S(t)I(t), \\ \dot{I}(t) = \beta\big(1 - I^{\sigma}(t)\big)S(t)I(t) - \gamma I(t). \end{cases} \tag{5}$$

The steady states of the system of differential equations are represented by solutions of the system of equations:

$$\begin{cases} (1 - I^{\sigma})SI = 0, \\ \beta(1 - I^{\sigma})SI - \gamma I = 0. \end{cases} \tag{6}$$

The system of differential equations (5) has an unlimited number of steady states (I^*, S^*) in the form: $\forall\phi \in [0,1]$ $(I^*,\ S^*) = (0, \phi)$. The states are located on a line segment with endpoints (0;0) and (0;1). The steady state (0;0) corresponds to such a population that (sooner or later) was entirely affected by the epidemic. The point (0;1) describes in turn a population that was not affected by any epidemic. In all other cases, the percentage of infected people equals $\phi \in (0,1)$.

In the analyses below, it was assumed that the average duration of infection is 14 days after which the infected patient recovers or dies. Then, in a discrete time (where t = 0, 1, … denotes subsequent days and t=1 denotes the initial day of infection), one obtains $\gamma = \frac{1}{14} \approx 0.071429$. Parameter β is selected so that the maximum percentage of infections is recorded after one year if no restrictions are imposed on social and economic life (i.e. at $\sigma \to +\infty$, in the original SIR model). Numerical simulations[5] carried out in discrete time show that the condition is met for β=0.10655. Hence, the authors used this value of the parameter β in the results of numerical simulations presented below. It was also assumed that $I_1 = 10^{-6}$, i.e. that one person per million people is infected on day 1 of the epidemic.

---

3 Integral R, representing a solution of differential equation (3), does not affect the growth paths of production Y and capital K in the economy. It is considered (principally) negligible in the discussion below.

4 An analytical implicit solution of the system of differential equations (5) in the original SIR model (i.e. at $\sigma \to +\infty$) can be found in the study by Harko, Lobo, Mak 2014. However, as the stability of steady states in the discussed system of differential equations is more interesting in the following sections than its analytical solution, this study ignored the solution proposed by Harko, Lobo and Mak.

5 Then, the differential equation (1) is replaced with an equivalent differential equation expressed by the formula: $\Delta S_t = -\beta\kappa_t S_{t-1} I_{t-1}$.

Figures 1 to 3 show simulated curves of social and economic activity indices κ and percentages of S, I and R at the values of the parameter σ equal 0.25, 1 and 2. Table 1 contains the minimum values of social and economic activity indices (κ_m), the minimum percentages of susceptible (S_m), the maximum percentages of infected (I_M), the maximum percentages of recovered and dead (R_M), the day during the epidemic when the number of infected people reached its peak value (T) and the accumulated percentage of deaths ($\sum_t D_t$) at various values of parameter σ.

The simulated curves of the analysed variables depicted in Figures 1 to 3 and in Table 1 demonstrate that:

- At σ=0.25, the maximum number of the infected was observed not on day 365 of the epidemic (as in a scenario of no restrictions on social and economic life), but as late as on day 535. Then, the percentage of those infected was around 5.6% (and not at 6.2% calculated for a scenario of no restrictions on social and economic life). After the initial 1,500 days of the epidemic, percentage S_m was around 77.3%, and the accumulated percentage of deaths (at a 3% index of mortality among the infected) reached around 6.8% (without any lockdown measures, the percentage values would have been at around 42.0% and 1.7%, respectively). The minimum value of the social and economic activity index in this case amounted to approximately 72.6%.

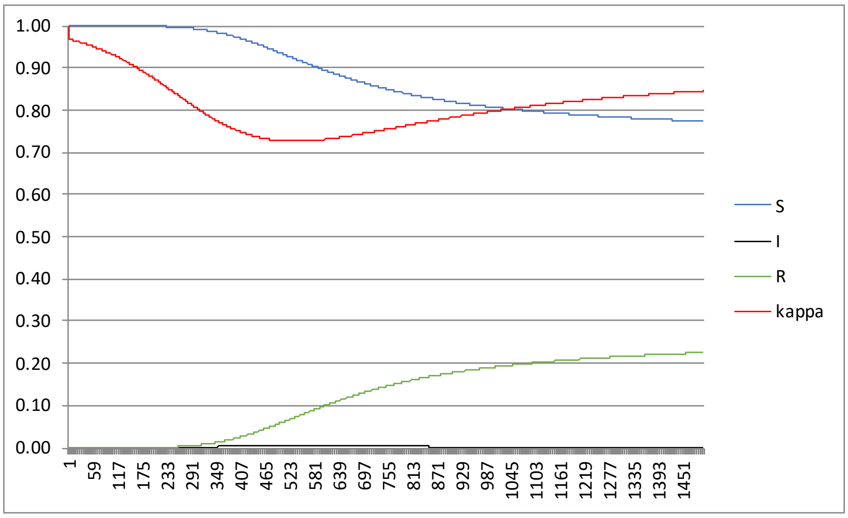

Fig. 1. Curves of the social and economic activity index κ and percentages of S, I and R at σ = 0.25 (days on the horizontal axis)

Source: own calculations.

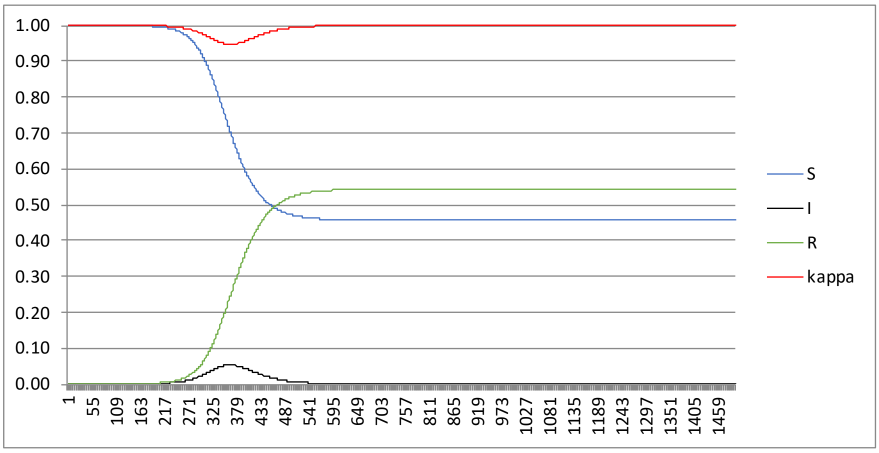

Fig. 2. Curves of the social and economic activity index κ and percentages of S, I and R at σ = 1 (days on the horizontal axis)

Source: own calculations.

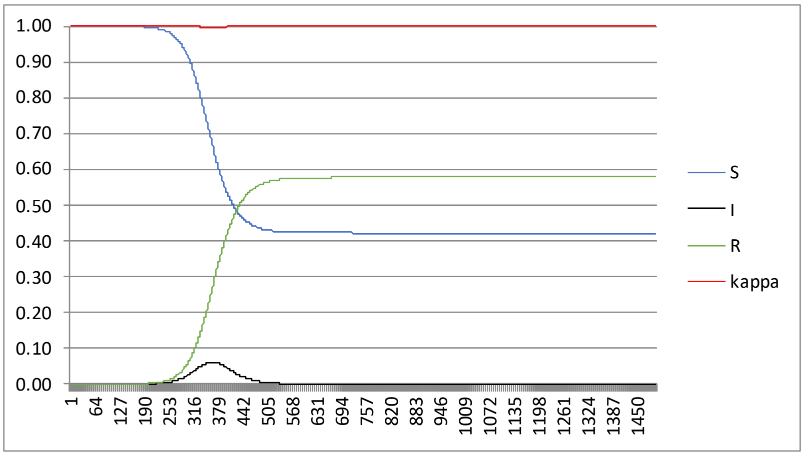

Fig. 3. Curves of the social and economic activity index κ and percentages of *S*, *I* and *R* at $\sigma = 2$ (days on the horizontal axis)

Source: own calculations.

- If $\sigma = 1$, the greatest number of infections was reached on day 366 of the epidemic at $I_M \approx 5.2\%$. After 1,500 days of the epidemic, 45.7% of the population did not catch the infection and the accumulated percentage of the dead was around 1.6%. On day 366 of the epidemic, social and economic activity κ fell by about 5.2% compared to a normal situation without an epidemic.
- If parameter σ equalled 2, the maximum percentage of the infected would have been observed on day 365 of the epidemic, when 6.2% of the population would be then infected. After 1,500 days, $S_m \approx 42.2\%$ and about 1.7% of the population died. At the time of peak incidence, social and economic activity was reduced only by approximately 4%.
- The results of the simulations in Table 1 demonstrate that if the severity of lockdown measures grows (which entails a fall in the value of parameter σ and thus a fall in the minimum index of social and economic activity κ_m), the percentage of susceptible S_m grows (at a falling percentage of I_M), the percentage of the recovered and dead R_M falls, day *T* when the maximum observed percentage of the infected is postponed, and the accumulated percentage of the dead $\sum_t D_t$ drops.

Table 1. The minimum value of the social and economic activity index (κ_m), the minimum percentage of susceptible (S_m), the maximum percentage of the infected (I_M), the maximum percentage of recovered and dead (R_M), the day of epidemic when the number of the infectious reaches its peak value (*T*), and the accumulated percentage of deaths ($\sum_t D_t$) at various values of parameter σ

σ	κ_m	S_m	I_M	R_M	T	$\sum_t D_t$[a]
0.1	0.672	0.999	0.0000	0.001	1775	0.00003
0.2	0.702	0.866	0.0023	0.133	653	0.00402
0.3	0.751	0.711	0.0097	0.289	473	0.00867
0.4	0.796	0.632	0.019	0.368	415	0.01105
0.5	0.835	0.577	0.027	0.423	390	0.01270
0.6	0.867	0.537	0.035	0.463	378	0.01390
0.7	0.894	0.507	0.041	0.493	372	0.01478
0.8	0.915	0.485	0.046	0.515	369	0.01544
0.9	0.933	0.469	0.050	0.531	367	0.01592
1	0.948	0.457	0.053	0.543	366	0.01629
2	0.996	0.423	0.062	0.578	365	0.01733
3	1.000	0.421	0.062	0.579	365	0.01738
4 and more	1.000	0.421	0.062	0.580	365	0.01739

Note: [a] – at a mortality rate among the infected at 3%.

Source: own calculations.

In the discussion below, the authors assumed that at σ=0.5 (and then *T*=390 and $\kappa_m \approx 0.835$) the state strongly responded to the epidemic, and at σ=1 (*T*=366 and $\kappa_m \approx 0.948$), and the government failed to take strong measures.

3.2. The economic module (a Solow-type model)

The macroeconomic module incorporates the following assumptions about both the production process (on the supply and demand side) and the capital accumulation process:

1) Assuming, just as Sahbani et al. (2019) proposed, that the value of production *Y* depends on the existing capital stock *K* and the percentage of susceptible[6] $1-I$. Thus, the effect of epidemic on supply in equation (7) is described by component $K^{\alpha}\big(\omega(1-I)\big)^{1-\alpha}$. The authors assumed (unlike the study indicated above) that the value of production *Y* is also affected by the index of social and economic activity κ on the supply side. Consequently, severe lockdown measures (accompanied by a low value of index κ) at given values of *K*, ω and $1-I$ lead to a reduced production value compared to a situation with less severe restrictions imposed on social and economic life. This is because, when the government imposes more severe lockdown restrictions, certain types of economic activity are officially limited, whilst incomes of people drop due to a reduced number of the employed. This leads to a drop in aggregate demand in the economy and (due to Keynesian multiplier effects) to a reduced level of utilisation of its production capacity. To simplify the analyses below, assume that the level of utilisation of production capacity of the economy (related to a drop in aggregate demand) is equal to the index of social and economic activity κ. Given the above assumptions, the Cobb-Douglas production function is expressed by the formula[7]:

$$Y(t) = \kappa(t)K^{\alpha}(t)\Big(\omega\big(1-I(t)\big)\Big)^{1-\alpha}, \tag{7}$$

where $\alpha, \omega \in (0,1)$. α denotes output elasticity of capital input (thus $1-\alpha$ denotes output elasticity of labour input), and ω denotes the working percentage of population under normal, non-epidemic conditions. This means that the working percentage of the population during an epidemic amounts to $\omega(1-I)$ instead of ω, as happens under normal non-epidemic conditions.

2) The capital accumulation equation (as in the Solow model) is expressed by the formula:

$$\dot{K}(t) = sY(t) - \delta K(t), \tag{8}$$

where $s, \delta \in (0,1)$; s denotes the rate of investment (the proportion of investment in production), while δ denotes the capital depreciation rate.

A differential equation is obtained from equations (7) and (8):

$$\dot{K}(t) = s\kappa(t)K^{\alpha}(t)\Big(\omega\big(1-I(t)\big)\Big)^{1-\alpha} - \delta K(t). \tag{9}$$

The study ignored the trivial solution *K*(*t*)=0 of differential equation (9), as uninteresting both from an economic and a mathematical point of view. Since $I \to 0^{+} \Rightarrow \kappa \to 1^{-}$, differential equation (9) is reduced to a differential equation:

6 The authors implicitly assumed that the percentage of the dead is too low to materially affect manufacturing processes. This is because the accumulated percentage of the dead does not exceed 1.63% of the population in the numerical simulations below. Assuming that the percentage is the same among the employed and that the output elasticity of labour input (i.e. $1-\alpha$) equals 0.4, the accumulated falls in production directly resulting from deaths will not exceed 6.52‰.

7 Production *Y* and capital stock *K* are expressed in a given currency at fixed prices.

$$\dot{K}(t) = s\omega^{1-\alpha}K^{\alpha}(t) - \delta K(t).$$

The above differential equation has exactly one nontrivial stable steady state K^* in which:

$$K^* = \omega\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

Stock K^*, by equation (7), corresponds to output Y^*:

$$Y^* = \omega\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

The quantities described by equations (10) and (11) correspond to capital stock K^*, and production Y^* in a long-term equilibrium determined by the original Solow model (with the Cobb-Douglas production function).

4. Calibration of model parameters and results of numerical simulations

As previously, assume that $\beta = 0.10655$ and $\gamma \approx 0.071429$. The key to the economic part of the model is output elasticity of capital input value (α). Epstein and Macchiarelli (2010) developed a methodology based on the production-function approach to estimate potential output of the Polish economy (for α = 0.558 – 0.514). Similarly, Vergos et al. (2010) investigated economies of scale in the Greek and Norwegian seafood sector. They used the Cobb-Douglas production function with the value of α=0.541, hence it was assumed that α=0.6. In this case, taking two economies with a capital-to-worker ratio k_1/k_2 of 3:1, then one obtains a work efficiency ratio of approximately 1.933 $\left(\frac{y_1}{y_2} = \left(\frac{k_1}{k_2}\right)^{\alpha} = 3^{0.6} \approx 1.933\right)$, which seems close to reality (whereas, if, in accordance with the so-called Solow's decomposition α=1/3, and k_1/k_2=3, one obtains $y_1/y_2 \approx 1.422$, which, in turn, is a much-underestimated value).

The authors also took that s=0.2, δ=0.05, ω=0.4 and $K(0)/K^* = 0.4$, and with the parameter values α, s and δ so calibrated, the long-term capital $K^* \approx 22.36$, and the long-term output $Y^* \approx 4.47$; see equations (10) and (11). This is why the value of long-term coefficient of capital intensity $v^* = \frac{K^*}{Y^*}$ is close to 5.

The following results of numerical simulations were obtained using four scenarios. The analyses tracked developments of the epidemic and economy under conditions of restrictions imposed by the government on social and economic life with continually growing severity as by formula $\kappa = 1 - I^{\sigma}$ (at values of parameter σ at 0.5 and 1). The authors also considered scenarios in which (as in the study by Bärwolff, 2020) the state makes an arbitrary decision on immediate lockdown. The index κ of social and economic activity is then described by the formula:

$$\kappa_t = \begin{cases} 1 & for \ \ \bar{I}_{tG(14)} < \iota, \\ \theta & for \ \ \bar{I}_{tG(14)} \geq \iota, \end{cases}$$

where $\theta, \iota \in (0,1)$, and $\bar{I}_{tG(14)} = \sqrt[14]{\prod_{i=1}^{14} I_{t-i}}$ denotes the 14-day geometric moving average of the percentage of infected I on day t. As per the above equation, if average $\bar{I}_{tG(14)}$ of the percentage of the infected is equal to or greater than a certain (arbitrarily determined) constant ι, the government will rapidly impose a lockdown, limiting the level of social and economic activity from 1 to θ.

The principal characteristics of the scenarios discussed below are given in Table 2.

Table 2. Scenarios of government's response to an epidemic

No.	Index κ of social and economic activity	Parameters	Scenario
1	$\kappa_t = 1 - I_{t-1}^{\sigma}$	σ=0.5, i.e. $\kappa_t = 1 - \sqrt{I_{t-1}}$	RS
2		σ=1, then $\kappa_t = 1 - I_{t-1}$	RW
3	$\kappa_t = \begin{cases} 1 & for\ \bar{I}_{tG(14)} < \iota \\ \theta & for\ \bar{I}_{tG(14)} \geq \iota \end{cases}$	$\theta = 0.85\ \wedge\ \iota = 10^{-5}$	AS
4		$\theta = 0.95\ \wedge\ \iota = 10^{-3}$	AW

Source: own studies.

Scenario RS is a scenario with severe restrictions, imposed in line with rule (4), while scenario RW with mild restrictions. In scenario AS, the government imposes a 15% lockdown rapidly when at least 1 person per 100 thousand inhabitants is infected (a severe lockdown), scenario AW includes a 5% lockdown in the case of $\iota \geq 1‰$ (a mild lockdown)[8]. The following system of difference equations is solved in scenarios RS and RW:

$$\begin{cases} \Delta S_t = -\beta(1 - I_{t-1}^{\sigma})S_{t-1}I_{t-1}, \\ \Delta I_t = \beta(1 - I_{t-1}^{\sigma})S_{t-1}I_{t-1} - \gamma I_{t-1}, \\ \Delta K_t = s\frac{Y_{t-1}}{365} - \delta\frac{K_{t-1}}{365}, \\ Y_t = (1 - I_t^{\sigma})K_t^{\alpha}\big(\omega(1 - I_t)\big)^{1-\alpha}, \end{cases} \tag{12}$$

and the system solved in scenarios AS and AW reads:

$$\begin{cases} \kappa_t = \begin{cases} 1\ for\ \bar{I}_{tG(14)} < \iota, \\ \theta\ for\ \bar{I}_{tG(14)} \geq \iota, \end{cases} \\ \Delta S_t = -\beta(1 - I_{t-1}^{\sigma})S_{t-1}I_{t-1}, \\ \Delta I_t = \beta(1 - I_{t-1}^{\sigma})S_{t-1}I_{t-1} - \gamma I_{t-1}, \\ \Delta K_t = s\frac{Y_{t-1}}{365} - \delta\frac{K_{t-1}}{365}, \\ Y_t = (1 - I_t^{\sigma})K_t^{\alpha}\big(\omega(1 - I_t)\big)^{1-\alpha}. \end{cases} \tag{13}$$

The numerical solutions of systems of equations (12 and 13) are used to draw curves that are analysed in the model of epidemiological and macroeconomic variables (Figures 4 to 8).

Figure 4 contains curves representing percentages of the infected in the analysed scenarios. The chart demonstrates that rapidly imposed restrictions on social and economic life postpone the day of the greatest percentage of the infected from day 390 to day 550 of the epidemic under conditions of a severe lockdown (curves RS and AS), or from day 366 to day 384 under mild lockdown conditions (curves RW and AW). Under conditions of severe restrictions imposed gradually, the maximum percentage of the infected reaches around 2.7%; under severe restrictions imposed rapidly – around 2.4%; under mild restrictions imposed gradually – around 5.3%, and under mild restrictions imposed rapidly – around 4.9%.

Figure 5 depicts curves representing accumulated percentages of the dead within 1,500 initial days of an epidemic. It demonstrates that the accumulated percentage of the dead becomes stable first in scenario RW (at a level of about 16.3%), then in scenario AW (15.8%), RS (12.7%), and last in scenario AW (11.7%).

[8] The scenarios RS (RW) are also referred to as scenarios of severe (mild) restrictions imposed gradually on social and economic life, and scenarios AS (AW) as scenarios of severe (mild) restrictions imposed rapidly on social and economic life.

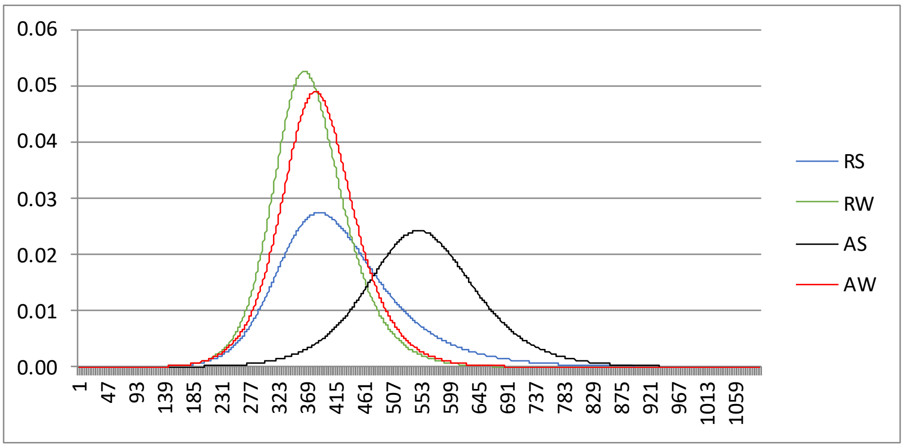

Fig. 4. Curves representing percentages of the infectious *I* in various lockdown scenarios (1,100 days on the horizontal axis)

Source: own calculations.

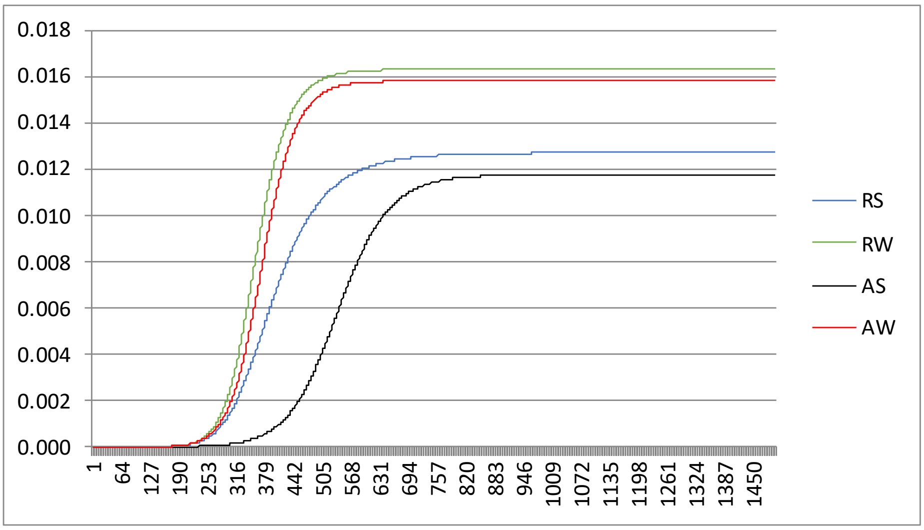

Fig. 5. Curves representing percentages of the dead $\sum_t D_t$ in various scenarios (1,500 days on the horizontal axis)

Source: own calculations.

The production curves obtained in numerical simulations are depicted in Figure 6, showing that:

- In each of the cases considered, an epidemic results in a (deep or mild) recession, demonstrated by macroeconomic indicators.
- If restrictions are gradually imposed and lifted, the curves of production are smooth; if restrictions are rapidly imposed and lifted, the curves are not smooth. In other words, the recession and recovery occur abruptly.
- In the scenario of severe restrictions gradually imposed on social and economic life (scenario RS), production will fall from month 3 to month 13 of the epidemic, reaching in month 13 a level of approximately 83.6% of its value in the first month of the epidemic. After five years, production in this scenario will rise by 5.7% (if no epidemic occurred, the value of this macroeconomic variable would rise by about 6.3%). The accumulated value of production over five years will be about 3.7% lower than the value that would be reached under normal conditions.
- Under conditions of mild restrictions imposed gradually on social and economic life (scenario RW), a recession begins in month 7 and lasts for five months. Production in month 12 will drop (compared to the beginning of the epidemic) by about 5.8%, to rise in month 60 by 6.2% compared

to the value in the first analysed month. The accumulated loss of production[9] over five years will reach around 0.7%.

- In the scenario of severe restrictions rapidly imposed on social and economic life (as in the scenario of severe restrictions imposed gradually by the state), the value of production drops from month 3 to month 13 of the epidemic. The difference between those scenarios lies in the curve of production that is smooth in scenario RS and not smooth in scenario AS. If severe restrictions are gradually imposed, production drops in month 3 (compared to month 2) by around 0.4‰, and if severe restrictions are rapidly imposed, by around 7.7%. In month 13 of the epidemic, the value of production in scenario AS is approximately 14.9% lower than before the epidemic, and after 60 months it is around 4.5% higher. Accumulated production losses suffered over five years in this scenario reach approximately 9.6%.
- If mild restrictions are rapidly imposed, the value of production drops in month 8 of the epidemic by around 4.2% (if mild restrictions are gradually imposed, the value of production drops by around 0.1%). The drop in production lasts for five months. In month 13, the value of that macroeconomic variable is around 6.7% lower than before the epidemic, and after five years it will around 6.0% higher than in month 1 of the epidemic. The accumulated production losses suffered over five years will reach approximately 1.4%.

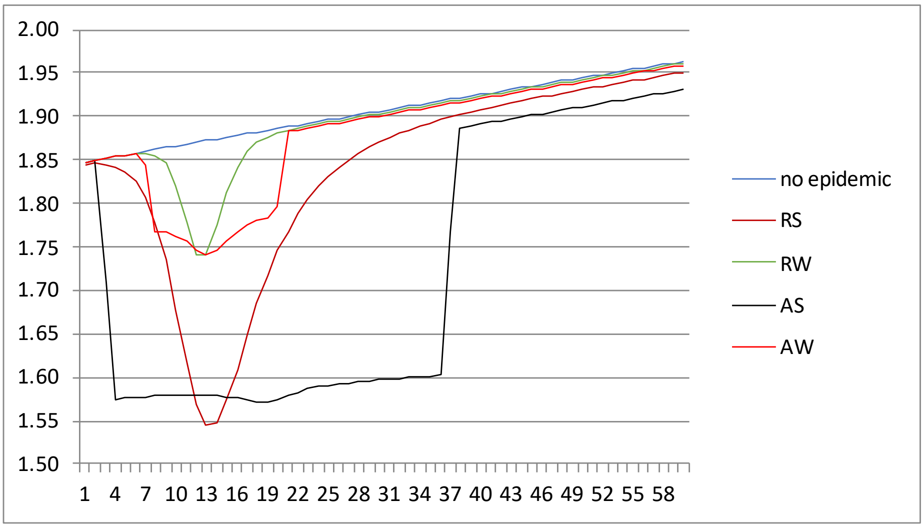

Fig. 6. Curves representing production *Y* under conditions without epidemic (blue) and in various lockdown scenarios (5 years, numbers of months are given on the horizontal axis)

Source: own calculations.

Figure 7 shows curves representing capital under normal conditions (blue) and in various scenarios of state response to the epidemic. It leads to the conclusion that under conditions of severe, rapidly imposed restriction the rate of capital accumulation significantly drops for more than three years. Under severe, gradually imposed restrictions, the downturn lasts for about 1.5 years. In the remaining scenarios (mild restrictions imposed on social and economic life), the capital curves are similar to those drawn for normal conditions.

In scenario RS, the accumulated capital losses will amount over five years to around 0.8%, amounting to around 0.2% in scenario RW, around 2.0% in scenario AS, and finally to approximately 0.3% in scenario AW. The significantly smaller accumulated capital losses, compared to the accumulated

[9] The loss is measured by the quotient$(\sum_{t=1}^{60} Y_{et})/(\sum_{t=1}^{60} Y_t)$, where Y_{et} denotes the value of production in month *t* of the epidemic, Y_t denotes the value of production that would be reached if no epidemic occurred. The accumulated capital losses are calculated using a similar method: $(\sum_{t=1}^{60} K_{et})/(\sum_{t=1}^{60} K_t)$.

production losses, in each of the analysed scenarios are explained by the fact that restrictions on social and economic life have a stronger effect on aggregate demand (and by Keynesian multiplier effects, on the value of production) than on the supply side of the economy and the capital accumulation process.

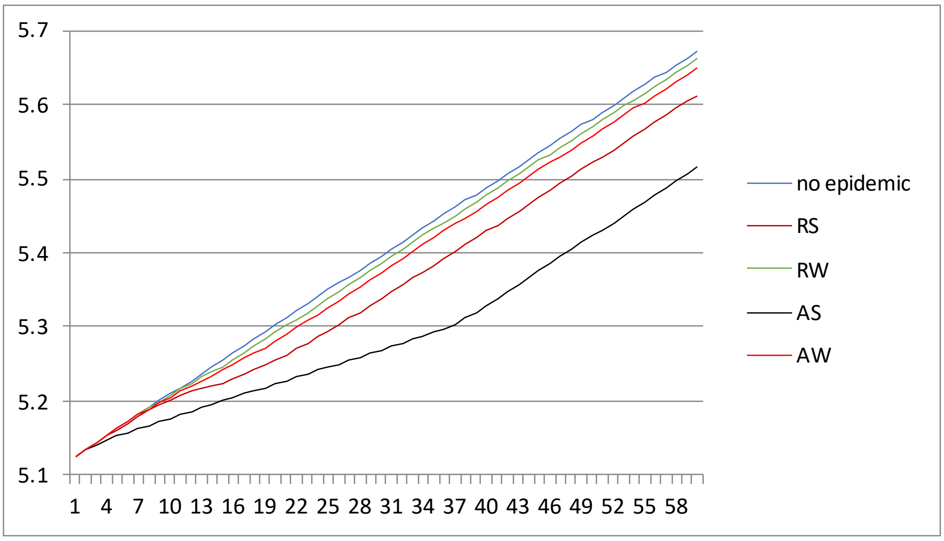

Fig. 7. Curves representing capital *K* under conditions without epidemic (blue) and in various lockdown scenarios (5 years, numbers of months on the horizontal axis)

Source: own calculations.

To assess the effect of an epidemic on the quality of social and economic life, the authors define the following function of social utility:

$$u_t = \sqrt[3]{(1 - I_t)\kappa_t \frac{c_{et}}{c_t}}. \tag{14}$$

The expression $\sqrt[3]{1 - I_t}$ in function (14) describes social utility understood as preventing infections in the population, $\sqrt[3]{\kappa_t}$ – utility resulting from the freedom of social contacts, and $\sqrt[3]{\frac{c_{et}}{c_t}}$ – utility resulting from the value of consumption reached under epidemic conditions compared to a value that would be reached under normal conditions, where $c_t = (1 - s)Y_t$; if not for the epidemic, on each day *t*, $1 - I_t = \kappa_t = \frac{c_{et}}{c_t} = 1$ and consequently also u_t = 1. Social utility described by equation (14) is interpreted so that if on day *t* it has the value of, e.g. 0.7, the aggregate social losses on that day (caused by infections, restrictions on social contacts and a reduced value of production) amount to 30%.

Figure 8 shows curves representing social utility in the analysed scenarios. As in the curves of consumption, restrictions gradually imposed and lifted result in smooth curves of social utility, while restrictions rapidly imposed and lifted result in curves that are not smooth. In scenario RS, the level of social utility drops during the peak of the epidemic by around 12.5%, and by around 5.9% in scenario RW, by around 11.5% in scenario AS, and by approximately 5.6% in scenario AW. The geometric average values of that economic variable reach approximately 97.5% under severe restrictions gradually imposed on social and economic life, 99.5% under mild, gradually imposed restrictions, 93.7% under severe, rapidly imposed restrictions, and around 99.0% under mild, rapidly imposed restrictions.

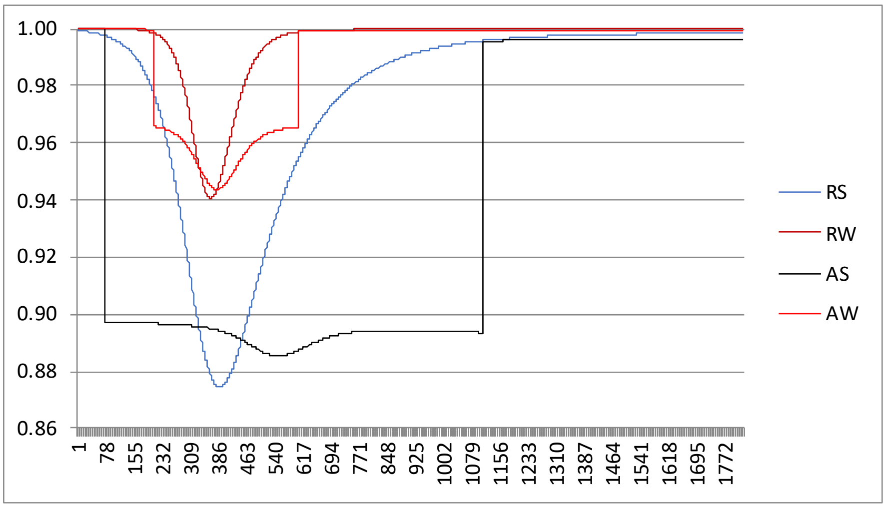

Fig. 8. Curves representing social utility $u_t = \sqrt[3]{(1-I_t)\kappa_t \frac{c_{et}}{c_t}}$ in various lockdown scenarios (5 years on the horizontal axis)

Source: own calculations.

Tables 3 to 5 contain selected epidemiological and macroeconomic indicators in the various scenarios analysed here. The indicators describe an economy with an input capital stock $K(0)$ worth 40% of its value in Solow's long-term equilibrium K^* (Table 3), 70% of K^* (Table 4) and economic conditions of Solow's long-term equilibrium (Table 5)[10].

Table 3. Selected epidemiological and macroeconomic indicators in various lockdown scenarios ($K(0)=0.4K^*$)

Indicator	Scenario			
	RS	RW	AS	AW
T (days)	390	366	550	384
Day one of lockdown			75	208
Last day of lockdown			1108	605
Lockdown duration (days)			1034	398
$\bar{\kappa}_G{}^a$	0.961	0.995	0.894	0.987
I_M	0.0274	0.0525	0.0242	0.0489
$\sum_t D_t$ ab	0.0127	0.0163	0.0117	0.0158
$\sum_t c_{et}/\sum_t c_t$ cd	0.963	0.993	0.904	0.986
$\min_t\{c_{et}\}/c_0$ cd	0.836	0.942	0.851	0.943
$\sum_t K_{et}/\sum_t K_t$ cd	0.992	0.998	0.980	0.997
$\bar{u}_G$ c	0.975	0.995	0.935	0.990
u_m c	0.874	0.941	0.885	0.944

Notes: a – for days t from 1 to 1500; b – at a mortality rate among the infected at 3%, c – over 5 years; d – monthly.

Indicators: T – the day of the greatest percentage of the infected, $\bar{\kappa}_G$ – geometric average of the social and economic activity index (from daily simulations), I_M – the maximum percentage of the infected, $\sum_t D_t$ – the accumulated percentage of the dead, $\sum_t c_{et}/\sum_t c_t$ the accumulated value of production during the epidemic compared to a value under conditions without epidemic, $\min_t\{c_{et}\}/c_0$ the minimum value of consumption compared to its input value (from monthly simulations), $\sum_t K_{et}/\sum_t K_t$ – accumulated capital stock during the epidemic compared to that achievable without epidemic, $\bar{u}_G$ – geometric average of social utility (from daily simulations), u_m – minimum daily social utility.

Source: own calculations.

[10] Tables 4 to5 do not contain epidemiological indicators because they do not depend on the input capital stock value $K(0)$.

Table 4. Selected epidemiological and macroeconomic indicators in various lockdown scenarios ($K(0)=0.7K^*$)

Indicator	Scenario			
	RS	RW	AS	AW
$\sum_t c_{et}/\sum_t c_t$ [ab]	0.964	0.993	0.905	0.986
$\min_t\{c_{et}\}/c_0$ [ab]	0.829	0.935	0.842	0.935
$\sum_t K_{et}/\sum_t K_t$ [ab]	0.993	0.999	0.984	0.997
$\bar{u}_G$ [a]	0.975	0.995	0.936	0.990
u_m [a]	0.874	0.941	0.885	0.944

Notes: [a] – over 5 years; [b] – monthly. Indicators as in Table 3.

Source: own calculations.

Table 5. Selected epidemiological and macroeconomic indicators in various lockdown scenarios ($K(0)=K^*$)

Indicator	Scenario			
	RS	RW	AS	AW
$\sum_t c_{et}/\sum_t c_t$ [ab]	0.964	0.994	0.906	0.986
$\min_t\{c_{et}\}/c_0$ [ab]	0.825	0.931	0.837	0.931
$\sum_t K_{et}/\sum_t K_t$ [ab]	0.994	0.999	0.986	0.998
$\bar{u}_G$ [a]	0.975	0.995	0.937	0.990
u_m [a]	0.875	0.941	0.885	0.944

Note: [a] – over 5 years; [b] – monthly. Indicators as in Table 3.

Source: own calculations.

A comparison of the indicators in Tables 3 to 5 describing the economic consequences of an epidemic leads to the conclusion that the differences in the analysed macroeconomic variables between the three economies are insignificant. Hence, an epidemic has a (relatively) similar effect on a poor economy (with $K(0)=0.4K^*$), on an economy with a medium level of development ($K(0)=0.7K^*$), and on a rich economy ($K(0)=K^*$).

5. Conclusion

The simulations made using the SIR model (i.e. the first module of this epidemiological-economic model) demonstrate that if the percentage of the infected rises from 0 to 1, the state by rapidly imposing lockdown restrictions causes a reduction in the social and economic activity index from 1 to 0. In a scenario with the value of parameter at σ=1, an increase in the percentage of the infected by 1% automatically provokes the government to impose restrictions, and results in a drop in the social and economic activity index by 1%. However, the characteristic of epidemic spread in the analysed society is an important factor. The spread in large urban centres is certainly different than in rural areas (this provides reasons for parallel simulations made for various values of parameter β in various regions that may be heterogeneously affected by the epidemic). The same restrictions on social and economic life imposed in the entire territory of a country may be fatally erroneous. The problem will be examined by the authors in the future.

Second, if the severity of lockdown measures grows (which entails a fall in the value of parameter σ and thus a fall in the minimum index of social and economic activity κ_m), the percentage of the susceptible grows (at a falling percentage of the infected), the percentage of the recovered and dead falls, the day when the maximum percentage of the infected is observed, is postponed, and the accumulated percentage of the dead decreases.

In the simulations made using the Solow model (the second module of the epidemiological-economic model applied), four scenarios of pandemic development and economy's reaction were analysed, considering various levels of restrictions imposed by the government using a fixed rule or in an arbitrary manner (*ad hoc*). Thus, under conditions of severe restrictions gradually imposed on social and economic life, the social and economic activity index fell after 390 days to around 83.5% of its level preceding the epidemic. Under conditions of mild restrictions gradually imposed over a period of 366 days, the social and economic activity index decreased by the end of that period to approximately 94.8%. If severe lockdown measures are rapidly implemented by the state, social and economic activity drops by 15% on day 75 of the epidemic; the lockdown measures then remain in force for 1,034 days. A mild 5% lockdown will rapidly be imposed on day 208 of the epidemic, and the restrictions will remain in force for 398 subsequent days.

The geometric average value of the social and economic activity index over 1,500 days of the epidemic will reach around 96.1% in a scenario of severe restrictions imposed in line with a rule, 99.5% in a scenario of mild restrictions, 89.4% in a scenario of rapid decisions to impose a severe lockdown, and 98.7% in a scenario of rapid decisions to impose a mild lockdown. The rapid introduction of lockdown measures has a stronger effect on accumulated social and economic activity than a continual process of imposing and lifting restrictions on that activity.

An analysis of changes in the percentage of the infections in the considered scenarios leads to the general conclusion that severe restrictions rapidly imposed on social and economic life postpone the day of the greatest percentage of the infected, compared to the scenario of severe, gradually imposed restrictions (namely from day 390 to day 550 of the epidemic). In the scenarios of mild restrictions imposed rapidly and gradually, the difference amounts to 18 days. The differences between the maximum values of the percentage of the infectious in adequately juxtaposed scenarios are at 0.3% for severe restrictions imposed by the government (gradually as a rule, compared to 15% for a rapid lockdown), and 0.4% for mild restrictions (imposed gradually as a the rule, compared to 5% for a rapid lockdown).

With the same assumptions retained, the curves of production distinctly differ in the analysed four scenarios. Reference conditions are provided by a situation without epidemic in which the existing rate of rise in the value of aggregate production is preserved. If severe restrictions are imposed gradually, as a rule production may fall from month 3 to month 13 of the epidemic, reaching in month 13 83.6% of its value in month one of the epidemic. After five years, production will grow in this scenario by 5.7%, which means that it will not reach the value of accumulated growth that would be achieved under normal conditions (no epidemic). Mild restrictions imposed gradually as a rule, will probably result in a shorter period of reduced production (five months), and in a similar accumulated growth rate over a period of five years as under normal conditions (no epidemic). A similar production curve is obtained in the scenario of a mild, rapidly imposed lockdown. The difference lies principally in the slower pace of return to the former growth curve. In the analysed scenarios, a rapid and severe lockdown has the most drastic consequences for the economy, considering both the value of drop in production, the time of remaining on the path of the relatively lowest growth and the expected final output. Expected accumulated production losses suffered over five years may reach around 1.4%.

The response of capital is significantly weaker under similar conditions, which naturally results from the characteristic of capital as a resource that (even under conditions of no replacement investments), responds in other ways than the production stream to epidemic conditions. In the scenario of severe, rapidly imposed restrictions, the rate of capital accumulation is significantly reduced for over three years. Under severe, gradually imposed restriction, the downturn lasts for a shorter period (1.5 years). In the remaining two scenarios of mild restrictions, the capital curves are close to those drawn for normal conditions.

The discussion contained in this study was enriched by an analysis of the effect of an epidemic on social utility understood as the quality of life, determined by the resistance to infection (prevention of infections in the population), social and mental wellbeing (related to restrictions on social contacts), and financial wellbeing (related to changes in the value of production). Attention is drawn to the strikingly similar curves of social utility, compared to the curves of changes in consumption in the same four scenarios with the retained framework conditions for analyses. Considering the sharp drop and its duration, the scenario of rapid severe lockdown may cause the most dramatic loss in social utility.

The study's theoretical analyses of the effect of an epidemic on three groups of economies (poor, on a medium level of development, and rich) lead to an initial hypothesis on the similar effect of an epidemic on economies on various levels of development, if the patterns of spread of the epidemic are similar. This conclusion is surprising in the authors' opinion, hence it requires further studies to be verified or falsified.

References

Alvarez, F., Argente, D., & Lippi, F. (2020). A simple planning problem for COVID-19 lockdown. *NBER Working Paper*, No. 26981. http://www.nber.org/papers/w26981

Aum, S., Lee, S., & Shin, Y. (2020). Inequality of fear and self-quarantine: Is there a trade-off between GDP and public health? *NBER Working Paper*, No. 27100. http://www.nber.org/papers/w27100

Arino J., & van den Driessche, P. (2003). A multi-city epidemic model. *Mathematical Population Studies*, 10, 175-193.

Atkeson, A. (2020). What will be the economic impact of COVID-19 in the US? Rough estimates of disease scenarios. *NBER Working Paper*, No. 26867. http://www.nber.org/papers/w26867

Bärwolff, G. (2020). Mathematical modeling and simulation of the COVID-19 pandemic. *Systems*, 8(3), 1-12. https://doi.org/10.3390/systems8030024

Bloom, D., & Lyons, J. (Eds.). Economic implications of AIDS in Asia, United Nations Development Programme, New York, NY (1993).

Bloom, D., & Mahal, A. (1995). Does the AIDS epidemic really threaten economic growth? *NBER Working Paper*, No. 5148, NBER, Cambridge, MA. https://doi.org/10.1016/S0304-4076(96)01808-8

Bloom, D., & Mahal, A. (1997). Does the AIDS epidemic really threaten economic growth? *Journal of Econometrics*, 77(1), 105-124.

Brock, W., & Xepapadeas, A. (2020). The economy, climate change and infectious diseases: Links and policy implications. *Environmental and Resource Economics*, 76, 811-824. https://doi.org/10.1007/s10640-020-00442-z

Coccia, M. (2021). The relation between length of lockdown, numbers of infected people and deaths of Covid-19, and economic growth of countries: lessons learned to cope with future pandemics similar to Covid-19 and to constrain the deterioration of the economic system. *Science of The Total Environment*, (775), 145801. https://doi.org/10.1016/j.scitotenv.2021.145801

Cuddington, J. (1993). Modeling the macroeconomic effects of AIDS, with an application to Tanzania. *The World Bank Economic Review*, 7, 173-789.

Cuddington, J., & Hancock, J. (1994). Assessing the impact of aids on the growth path of the Malawian economy. *Journal of Development Economics*, 43(2), 363-368.

Cuesta, J. (2010). How much of a threat to economic growth is a mature AIDS epidemic? *Applied Economics*, 42(24), 3077-3089.

Delfino, D., & Simmons, P. (2005). Dynamics of tuberculosis and economic growth. *Environment and Development Economics*, 10(6), 719-743. doi:10.1017/S1355770X05002500

Epstein, N. P., & Macchiarelli, C. (2010). Estimating Poland's potential output: A production function approach. *IMF Working Paper*, WP/10/15.

Fei-Ying, Y., Wan-Tong, L., & Zhi-Cheng, W. (2015). Traveling waves in a nonlocal dispersal SIR epidemic model, *Nonlinear Analysis: Real World Applications*, 23, 129-147.

Harko, T., Lobo, F. S. N., & Mak, M. K. (2014). Exact analytical solutions of the Susceptible-Infected-Recovered (SIR) epidemic model and the SIR model with equal death and birth rates. *Applied Mathematics and Computation*, 236, 184-194.

Jardón-Kojakhmetov, H., Kuehn, Ch., Pugliese, A., & Sensi, M. (2021). A geometric analysis of the SIR, SIRS and SIRWS epidemiological models. *Nonlinear Analysis: Real World Applications*. 58, 103220.

Kambou, G., Devarajan, S., & Over, M. (1992). The economic impact of AIDS in an African country: Simulations with a computable general equilibrium model of Cameroon. *Journal of African Economies*, 1, 109-130.

Kermack, W. O., & McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society,* Ser. A, 115, 700-721.

Lik Ng, W. (2020). To lockdown? When to peak? Will there be an end? A macroeconomic analysis on COVID-19 epidemic in the United States. *Journal of Macroeconomics*, 65, 103230.

Lovasz, E., & Schipp, B. (2009). The impact of HIV/AIDS on economic growth in Sub-Saharan Africa. *South African Journal of Economics*, 77(2), 245-256.

Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, 107(2), 407-437.

McDonald, S., & Roberts, J. (2006). AIDS and economic growth: A human capital approach. *Journal of Development Economics*, 80(1), 228-250.

Murray, J. D. (2003). Mathematical biology, II, Spatial models and biomedical applications, third ed., In *Interdisciplinary applied mathematics*, vol. 18, Springer-Verlag.

Pedauga, L., Sáez, F., & Delgado-Márquez, B. L. (2022). Macroeconomic lockdown and SMEs: The impact of the COVID-19 pandemic in Spain. *Small Business Economics*, 58, 665-688. https://doi.org/10.1007/s11187-021-00476-7

Ruan, S. (2007). Spatial-temporal dynamics in nonlocal epidemiological models, In Y. Takeuchi, K. Sato, Y. Iwasa (Eds.), *Mathematics for life science and medicine*, Springer-Verlag, pp. 99-122.

Ruan, S., Wu, J. (2009). Modeling spatial spread of communicable diseases involving animal hosts, *In Spatial ecology*, Chapman & Hall/CRC, pp. 293-316.

Sahbani, S., Talibi Alaoui, H., & Kadar, A. (2019). Fluctuations of Solow economic growth model in the presence of an infectious disease. *Applied Mathematical Sciences*, 13(13). 627-636.

Sattenspiel, L., & Dietz, K. (1995). A structured epidemic model incorporating geographic mobility among regions. *Mathematical Biosciences*, 128, 71-91.

Solow, R. M. (1956). A Contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70(1), 65-94.

Vergos, K., Christopoulos, A., Krystallidis, P., & Papandroni, O. (2010). Economies of scale and concentration in the Greek and the Norwegian aquaculture industry: An empirical study. *International Journal of Business Management and Economic Research*, 1(1), 70-78.

Wang, Q., & Zhang, F. (2021). What does China's economic recovery after COVID-19 pandemic mean for the economic growth and energy consumption of other countries? *Journal of Cleaner Production*, 295, 126265, https://doi.org/10.1016/j.jclepro.2021.126265

Xiao, D., & Ruan, S. (2007), Global analysis of an epidemic model with nonmonotone incidence rate. *Mathematical Biosciences*, 208, 419-429.

Received: July 2022