Argumenta Oeconomica
Year 2025, Vol. 54, No. 1
ISSN 2720-5088
journals.ue.wroc.pl/aoe

Visualisation of linear ordering results using multidimensional scaling – problems and an overview of studies

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Quote as: Walesiak, M., Dehnel, G., & Dudek, A. (2025). Visualisation of linear ordering results using multidimensional scaling – problems and an overview of studies. *Argumenta Oeconomica*, *1*(54). 187-203.

DOI: 10.15611/aoe.2025.1.12

JEL: C38, C43, C63, Q01, Q43

Abstract

Aim: The aim of the article was to review the methodological solutions proposed as part of the hybrid method, which combines linear ordering with multidimensional scaling for various types of data. In the first step, after applying multidimensional scaling, it was possible to visualise objects of interest in a two-dimensional space. In the second step, the objects were linearly ordered according to an aggregate measure based on the Euclidean distance.

Methodology: The general procedure of the hybrid method, which can be used to visualise results of linear ordering for metric, ordinal and interval-valued data, was presented.

Results: The authors highlight the problems associated with the use of multidimensional scaling in linear ordering and how they can be solved. These problems with the application of multidimensional scaling in linear ordering are illustrated by an attempt to rank 27 EU countries in 2021 according to their progress towards reaching the sustainable development goal (SDG7). The article also contains an overview of studies involving the hybrid method.

Implications and recommendations: If the distribution of errors related to the arrangement of individual objects in the scaling space (stress-per-point values – *spp*) deviates significantly from the uniform distribution, the ranking of objects based on the results of multidimensional scaling is distorted. To solve this, the paper proposes to select the optimal multidimensional scaling procedure considering two criteria: Kruskal's goodness-of-fit statistic and the Herfindahl-Hirschman index, calculated using *spp* values. The use of the hybrid method is facilitated by the mdsOpt package in R environment.

Originality/value: The hybrid method makes use of the concept of isoquants and the path of development (the shortest line connecting the pattern and anti-pattern of development) proposed by Hellwig (1981). By applying multidimensional scaling one can visualise the results of linear ordering for more than two variables, whereas other linear ordering methods cannot be used to visualise these results.

Keywords: aggregate measures, multidimensional scaling, R environment, mdsOpt package, hybrid method

1. Introduction and motivation

1.1. An overview of linear ordering methodology

The purpose of methods enabling a linear ordering of a set of objects is to rank objects according to a specific criterion. These methods can therefore be used when a criterion (a complex phenomenon) is chosen that cannot be measured directly (the so-called latent variable – see Paruolo et al., 2013). This criterion is then used to order objects from 'the best' to 'the worst'. A complex phenomenon of interest can be described by a set of preference variables. The method of linear ordering requires that these variables can at least be measured on an ordinal scale (a ranking can only be created if variable values can be ordered in terms of their magnitude). All the methods used for ordering a set of objects rely on a function that aggregates partial information about individual variables. Elements of a set of objects are ordered in terms of the magnitude of the aggregate measure. In the literature the term 'aggregate measures' is used interchangeably with composite indicators (Nardo et al., 2005; Saisana et al., 2005; Saltelli, 2007; El Gibari et al., 2019), synthetic indicators (Maggino, 2017), synthetic indices (Becker et al., 2017), and composite indices (Mazziotta, & Pareto, 2016; Greco et al., 2019).

The need to order a set of objects based on aggregate measures appears in various research areas, such as sustainable development, population ageing, innovation, the quality of life, health and services, social wellbeing, competitiveness and branding, social cohesion, tourist attractiveness, customer satisfaction, poverty, and social exclusion.

Many approaches to the construction of aggregate measures have been proposed in the literature. Taking into account the degree of compensation, three types of techniques are distinguished (El Gibari et al., 2019, p. 3): compensatory, partially compensatory, non-compensatory. In the compensatory approach, it is assumed that a poor score of one indicator can be offset by a high score on another indicator and, as a result, may not be reflected in the aggregated score of a composite indicator (Banihabib et al., 2017). In partially compensatory methods, an aggregate measure is constructed in such a way as to limit the impact of the compensation effect. The goal of non-compensatory methods is to develop an ordering algorithm that is more consistent than linear aggregation rules where no compensation between indicators is allowed. As a result, "all the weights reflect the relative importance of each indicator rather than a trade-off ratio" (El Gibari et al., 2019, p. 15). This approach is used in the literature on multi-criteria decision-making (MCDM) (see Munda, 2008; Munda, & Nardo, 2009). This article focuses on compensatory and partially compensatory aggregate measures.

Various concepts and solutions have been proposed to rank a set of objects (cf. Walesiak, & Dehnel, 2022; Walesiak et al., 2024):

- the concept of a development pattern (ideal point; the upper pole of development) and a measure of development were proposed by Hellwig (1968; 1972), who also introduced the concepts of a stimulant and a destimulant, the measure of development defined as a distance of an object from the development pattern;
- aggregate measures that take into account the pattern and the anti-pattern (anti-ideal point; the lower pole) of development (Hellwig, 1981; TOPSIS measure – Hwang, & Yoon, 1981);
- methods proposed in the 1970s and 80s in which measures of central tendency (the arithmetic mean, the geometric mean, the median) were used to construct aggregate measures. Various modifications were later introduced in order to, for example, include a method for normalising values of variables, introduce nominants to the set of variables, propose weights for variables, suggest other ways of constructing aggregate measures (cf. Borys et al., 1990);
- an aggregate measure for ordinal data using the generalised distance measure (GDM2) to gauge the distance of objects from the pattern of development (Walesiak, 1993; 1999);
- an aggregate measure based on special cases of a generalised mean (or the power mean of order *r*): minimum, harmonic mean, geometric mean, arithmetic mean, quadratic mean, cubic mean, maximum (Mazziotta, & Pareto, 2022);
- aggregate measures for fuzzy numbers: e.g. fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) (Chen, 2000); Hellwig's synthetic measure for triangular fuzzy numbers (Jefmański, & Dudek, 2016); interval-valued intuitionistic fuzzy synthetic measure (I-VIFSM) based on Hellwig's approach (Roszkowska, & Jefmański, 2021);
- an aggregate measure of development for interval-valued data (Młodak, 2014; interval-valued TOPSIS – Fu et al., 2020);
- an aggregate measure that accounts for spatial dependence (Antczak, 2013; Pietrzak, 2014);
- aggregate measures with a penalty function (Mazziotta, & Pareto, 2016; 2018);
- aggregate measures with an adjustment to the surroundings of a given object (Łysoń et al., 2016);
- the application of principal component analysis (PCA) to produce a linear ordering of objects based on the value of the first principal component (Bąk, 2018; Perkal, 1967);
- the static approach to relative taxonomy: the classic version proposed by Wydymus (2013) and the positional version proposed by Lira (2015);
- the dynamic approach to relative taxonomy: for classic data proposed by Walesiak and Dehnel (2022) and for interval-valued data, proposed by Walesiak and Dehnel (2023);
- an iterative approach to ranking a set of objects, whereby in each iteration the highest ranked object receives the next position in the ranking and is eliminated from the set of objects (Sokołowski, & Markowska, 2017);
- flexible linear ordering (Sokołowski, & Markowska, 2019).

In this article the authors discuss methods which combine multidimensional scaling with linear ordering for classical data (Walesiak, 2016) and interval-valued data (Walesiak, & Dehnel, 2018; Dehnel, & Walesiak, 2019; Walesiak, & Dehnel, 2020). In the case of classic data, presented in the form of a data matrix or a data cube, each variable describing an object is expressed by only one real number (metric data) or one category (ordinal data). For interval-valued data, presented in the form of a data table, each variable describing an object is expressed by a numerical interval.

1.2. The purpose of the article

The aim of the article was to review the methodological solutions proposed as part of the hybrid method, which combines linear ordering with multidimensional scaling for various types of data (metric, ordinal, interval-valued). The reviewed studies were analysed in terms of the usefulness of particular solutions, taking into account their possible applications (the solutions are compared in terms of data type: metric, ordinal, interval-valued; the approach used: static, dynamic; data source: primary, secondary; the research problem), as well as the problems associated with the use of multidimensional scaling in linear ordering.

In 2016, Walesiak proposed a two-step research procedure (the hybrid method), which can be used to visualise the results of linear ordering for metric data. In the first step, after applying multidimensional scaling, objects of interest can be visualised in two-dimensional space, whilst in the second step the objects are linearly ordered according to Hellwig's (1981) aggregate measure based on the Euclidean distance. The hybrid method makes use of the concept of isoquants and the path of development (the shortest line connecting the pattern and anti-pattern of development), which were proposed by Hellwig (1981). Using this method, it is possible to visualise the results of linear ordering for two variables, and by applying multidimensional scaling one can view the results of linear ordering for $m > 2$ variables.

Another article (Walesiak, 2017b) presented a modification of the hybrid method for ordinal data. The mdsOpt package, which can facilitate the selection of the optimal multidimensional scaling procedure for metric data was described by Walesiak and Dudek (2017). Walesiak and Dehnel (2018) proposed a modification of the hybrid method for interval-valued data. The mdsOpt package for selecting the optimal multidimensional scaling procedure for metric and interval-valued data was described in (Walesiak, & Dudek, 2020).

2. The hybrid method procedure

The general procedure of the hybrid method which can be used to visualise the results of linear ordering for various types of data (metric, ordinal, interval-valued), consists of the following steps (the description is based on the articles mentioned in Section 1.2):

1. Select a complex phenomenon that cannot be measured directly (e.g. the level of social cohesion).
2. Select a set of objects and a set of variables (metric, ordinal, interval-valued) closely related with the complex phenomenon of interest. These variables can be divided into three types of preference variables (stimulants, destimulants, nominants – formal definitions can be found e.g. in Hellwig (1981, p. 48) and Borys (1984, p. 118)). A pattern object and an anti-pattern object are added to the set of objects. Owing to the structure of the anti-pattern object, nominants need to be converted into stimulants. Coordinates of the pattern object represent the most favourable values of preference variables (maximum values for stimulants and minimum values for destimulants), while the coordinates of the anti-pattern object represent the least favourable values (minimum values for stimulants and maximum values for destimulants). In the case of interval-valued variables, the coordinates of the pattern and anti-pattern object are determined separately for the lower and upper values of the interval. The scale for measuring the ordinal variables is strengthened by the application of the method proposed by Walesiak (2014), based on the GDM2 distance (Walesiak, 1999), which is appropriate for ordinal data. As a result of transforming ordinal variables into metric variables, destimulants and nominants are transformed into stimulants (see Walesiak 2017b). The ordinalToMetric function for this purpose is available in the clusterSim package (Walesiak, & Dudek 2023a).
3. Combine the data in the form of:
   a. data matrix $\boldsymbol{X} = [x_{ij}]_{nxm}$ in the case of metric data (x_{ij} – the value of *j*-th variable for *i*-th object),

b. data table $\boldsymbol{X} = \left[x_{ij}^l, x_{ij}^u\right]_{nxm}$ (where $x_{ij}^l \leq x_{ij}^u$; x_{ij}^l (x_{ij}^u) – the lower (upper) bound of interval) in the case of interval-valued variables.

4. Normalise metric and interval-valued variables and arrange the data in the form of:

   a. normalised data matrix $\boldsymbol{Z} = [z_{ij}]_{nxm}$ in the case of metric data (z_{ij} – the normalised value of *j*-th variable for *i*-th object),
   b. normalised data table $\boldsymbol{Z} = \left[z_{ij}^l, z_{ij}^u\right]_{nxm}$, where $z_{ij}^l \leq z_{ij}^u$, z_{ij}^l (z_{ij}^u) – the normalised lower (upper) bound of the interval.

In the clusterSim package, eighteen normalisation methods are presented, which are denoted by symbols such as: n1, n2, n3, n3a, n4, n5, n5a, n6, n6a, n7, n8, n9, n9a, n10, n11, n12, n12a, n13 for the type argument of the data-normalisation function (see e.g. Walesiak, & Dudek, 2017). Since some of these methods yield identical results of multidimensional scaling, methods implemented in the clusterSim package are listed below and denoted:

a. n1, n2, n3, n5, n5a, n8, n9, n9a, n11, n12a – when all variables are ratio variables,
b. n1, n2, n3, n5, n5a, n12a – when at least one of the variables is an interval variable.

Interval-valued data require a special way of normalisation. The lower and upper interval limits of *j*-th variable for *n* objects are combined into one vector containing 2*n* observations. In this way it is possible to use normalisation methods that are appropriate for metric data. Metric variables are normalised using the data. Normalization function, while interval-valued variables by applying the interval_normalization function, both of which are available in the clusterSim package.

5. Choose a measure of distance for metric data (Manhattan, Euclidean, squared Euclidean, Chebyshev, GDM1 – see e.g. Everitt et al. 2011, pp. 49-50; Jajuga et al. 2003), calculate distances and create a distance matrix $\delta = [\delta_{ik}(Z)]_{nxn}$ $(i, k = 1, \dots, n)$. For interval-valued data select a measure of distance from Table 1 (e.g. Ichino-Yaguchi, Euclidean Ichino-Yaguchi, Hausdorff, Euclidean Hausdorff), calculate distances and create a distance matrix $\delta = [\delta_{ik}(Z)]_{nxn}$.

Table 1. Distance measures for interval-valued data

Symbol	Name	Distance measure $\delta_{ik}(Z)$
U_2_q1	Ichino-Yaguchi $q = 1, \gamma = 0{,}5$	$\sum_{j=1}^{m} \varphi(z_{ij}, z_{kj})$
U_2_q2	Euclidean Ichino-Yaguchi $q = 2, \gamma = 0{,}5$	$\sqrt{\sum_{j=1}^{m} \varphi(z_{ij}, z_{kj})^2}$
H_q1	Hausdorff $q = 1$	$\sum_{j=1}^{m} [\max(\lvert z_{ij}^l - z_{kj}^l\rvert, \lvert z_{ij}^u - z_{kj}^u\rvert)]$
H_q2	Euclidean Hausdorff $q = 2$	$\left\{\sum_{j=1}^{m} [\max(\lvert z_{ij}^l - z_{kj}^l\rvert, \lvert z_{ij}^u - z_{kj}^u\rvert)]^2\right\}^{1/2}$

Note: $z_{ij} = [z_{ij}^l, z_{ij}^u]$; $\varphi(z_{ij}, z_{kj}) = |z_{ij} \oplus z_{kj}| - |z_{ij} \otimes z_{kj}| + \gamma(2 \cdot |z_{ij} \otimes z_{kj}| - |z_{ij}| - |z_{kj}|)$; $|\cdot|$ – interval length; $z_{ij} \oplus z_{kj} = z_{ij} \cup z_{kj}$; $z_{ij} \otimes z_{kj} = z_{ij} \cap z_{kj}$.

Source: own presentation based on Billard, & Diday (2006), pp. 244-246; Esposito et al. (2000), pp. 165-185; Ichino, & Yaguchi (1994).

6. Perform multidimensional scaling (MDS): $f: \delta_{ik}(Z) \rightarrow d_{ik}(V)$ for all pairs (i, k), where *f* denotes distance mapping from *m*-dimensional space $\delta_{ik}(Z)$ into corresponding distances $d_{ik}(\boldsymbol{V})$ in a *q*-dimensional space ($q = 2$). The operation is performed using the smacofSym function from the smacof package (Mair et al., 2022).
7. Finally, as a result of applying multidimensional scaling, one obtains a two-dimensional data matrix $V = [v_{ij}]_{nxq}$. Depending on the location of the pattern and anti-pattern object in the two-dimensional scaling space $V = [v_{ij}]_{nx2}$ the coordinate system needs to be rotated by angle φ according to the formula:

$$[v'_{ij}]_{nx2} = [v_{ij}]_{nx2} \times D \quad (1)$$

where $[v'_{ij}]_{nx2}$ – data matrix in a two-dimensional scaling space after rotating the coordinate system by angle φ, $D = \begin{bmatrix} cos\varphi & -sin\varphi \\ sin\varphi & cos\varphi \end{bmatrix}$ – rotation matrix.

The rotation does not change the arrangement of objects relative to one another but makes it possible to position the set axis connecting the pattern and anti-pattern along the identity line, which improves the visualisation of results.

8. Visualise and interpret the results (of multidimensional scaling) in a two-dimensional space. This is done by:

- joining two points representing the pattern and anti-pattern by a straight line to form what is known as the set axis in the diagram,
- drawing isoquants of development (curves of equal development) from the pattern. Objects located between the isoquants represent a similar level of development. The same level can be achieved by objects located at different points along the same isoquant of development (due to a different configuration of variable values).

9. Order objects according to the value of aggregate measure d_i based on the Euclidean distance from the pattern object (Hellwig, 1981):

$$d_i = 1 - \sqrt{\sum_{j=1}^{2}(v_{ij} - v_{+j})^2} \Big/ \sqrt{\sum_{j=1}^{2}(v_{+j} - v_{-j})^2} \quad (2)$$

where v_{ij} – *j*-th coordinate of *i*-th object in the two-dimensional MDS, $v_{+j}(v_{-j})$ – *j*-th coordinate of the pattern (anti-pattern) in the two-dimensional MDS.

The values of aggregate measure d_i belong to the interval $[0; 1]$. The higher the value of d_i, the higher the level of development (e.g. in terms of social cohesion) of a given object. The objects are arranged according to descending values of aggregate measured_i.

3. Problems associated with the use of MDS in linear ordering

The monograph (Borg et al., 2018, chapter 7) pointed out the typical mistakes made by users of MDS; one frequent mistake is connected with the evaluation of stress (the goodness-of-fit statistic), which leads to rejecting an MDS solution because its stress is 'too high'. According to Borg et al. (2018, pp. 85-86) "The stress value is, however, merely a technical index, a target criterion for an optimization algorithm. An MDS solution can be robust and replicable, even if its stress value is high", and "stress is a *summative* index for *all* proximities. It does not inform the user how well a *particular* proximity value is represented in the given MDS space (...) The least one can do is to take a look at the stress-per-point values".

In light of the above, one should take into account values of stress-per-point (Borg, & Mair, 2017) and the Shepard diagram (Mair et al., 2016; De Leeuw, & Mair, 2015).

If the distribution of errors related to the arrangement of individual objects in the scaling space (stress-per-point values) deviates significantly from the uniform distribution (e.g. the sum of errors for several objects exceeds 40%, a relatively large error associated with one object, the pattern or the anti-pattern), this leads to an incorrect arrangement of objects on the plane:

- objects that should be closer to the pattern are closer to the anti-pattern,
- some objects may be located above the pattern or below the anti-pattern,

- the ranking of objects based on the results of multidimensional scaling in a two-dimensional space is distorted and does not reflect the actual situation.

To solve the problem of choosing the optimal MDS procedure, two criteria were applied in the mdsOpt package (Walesiak, & Dudek, 2023b):

- Kruskal's $Stress$-1 (standardised residual sum of squares) measure of fit (see e.g. Borg et al. 2018, p. 32):

$$Stress\text{-}1_p = \sqrt{\sum_{i<k} \left[d_{ik}(V) - \hat{d}_{ik}\right]^2 / \sum_{i<k} d_{ik}^2(V)}, \quad (3)$$

where p – MDS procedure number, $\hat{d}_{ik}$ – d-hats, disparities, target distances or pseudo distances (see Borg, & Groenen 2005, p. 199), $\hat{d}_{ik} = f(\delta_{ik})$ by defining f in different ways (ratio, interval, polynomial MDS).

- the Herfindahl-Hirschman HHI index (Herfindahl, 1950; Hirschman, 1964), calculated using stress-per-point values (*spp*):

$$HHI_p = \sum_{i=1}^{n} spp_{pi}^2, \quad (4)$$

where $i = 1, \ldots, n$ – object number, p – MDS procedure number.

The HHI_p index takes values from the interval $\left[\frac{10{,}000}{n}; 10{,}000\right]$. The value $\frac{10{,}000}{n}$ means that the distribution of errors for individual objects is uniform. The optimal situation for an MDS procedure is the minimum value of the HHI_p index.

The mdsOpt package enables users to select the optimal multidimensional scaling procedure by offering various normalisation methods, distance measures and scaling models (ratio, interval, mspline), which are implemented in the findOptimalSmacofSym function for classical data, and in the optSmacofSymInterval function for interval-valued data (Walesiak, & Dudek, 2023b).

For all MDS procedures, for which $Stress - 1_p \leq cs$ (cs – a midrange of $Stress$-1), the authors chose the one where $\min_p\{HHI_p\}$ is reached.

The problems with the application of multidimensional scaling in linear ordering were illustrated by an attempt to rank 27 EU countries in 2021 according to their progress towards reaching the sustainable development goal (SDG7), i.e. "Ensure access to affordable, reliable, sustainable and modern energy for all". Since 2017, Eurostat has published an "Annual EU SDG indicator review" (https://ec.europa.eu/eurostat/web/sdi/information-data), which contains an updated list of indicators for 17 SDGs (SDGs in the EU context). Seven indicators, defined in 2017 to monitor progress on SDG7, have not changed until now (see Table 2).

Table 2. Indicators for SDG7 for EU countries

Headline indicators	Variable type	Unit
Primary energy consumption	D	mtoe
		2005 = 100
Final energy consumption	D	mtoe
		2005 = 100
Final energy consumption in households per capita	D	kgoe
Energy productivity	S	Euro per kgoe
Share of renewable energy in gross final energy consumption	S	%
Energy import dependency	D	%
Share of population unable to keep home adequately warm	D	%

Note: S – stimulants (where higher values are more preferred), D – destimulants (where lower values are more preferred).

Source: authors' compilation based on Eurostat (2023).

Ten normalisations methods (implemented in the data.Normalization function from the clusterSim package: n1, n2, n3, n5, n5a, n8, n9, n9a, n11, n12a), four distance measures (Manhattan, Euclidean, squared Euclidean, GDM1) and four MDS models (ratio, interval, mspline model – polynomial function of the second and third degree) were used for selecting the optimal MDS procedure.

Figure 1 shows the relationship between $Stress$-1 and the HHI index, with the best solution denoted by the red circle, which satisfies the condition $Stress - 1 \leq$ a midrange of $Stress - 1$ and minimises HHI.

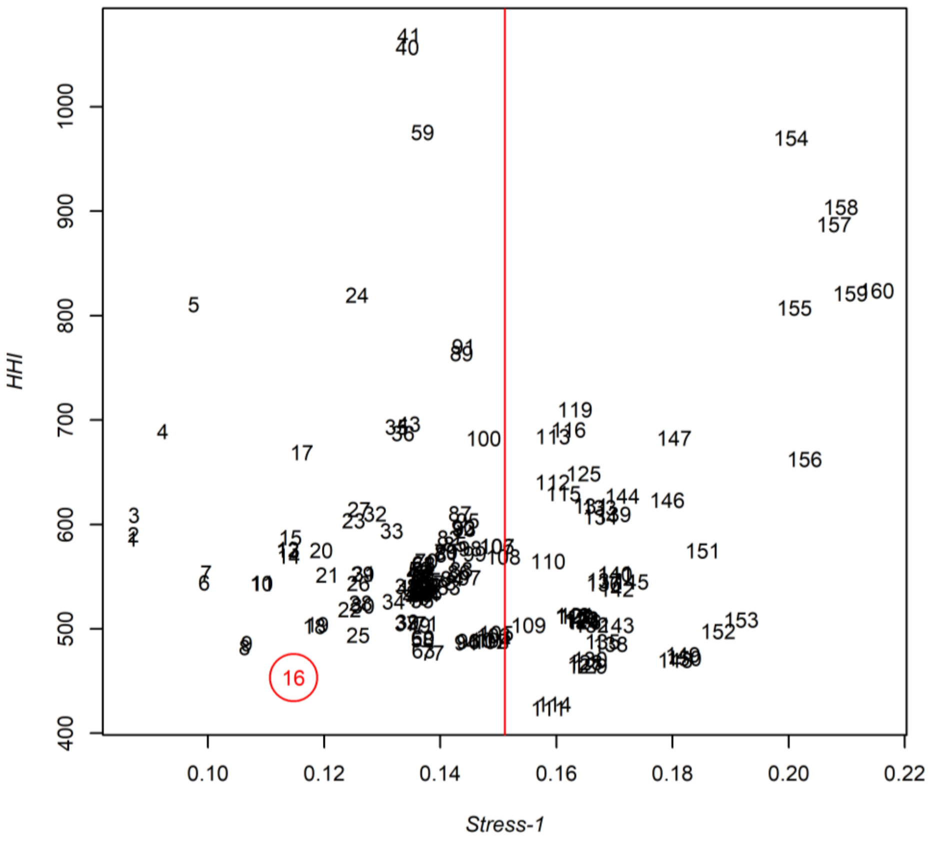

Fig. 1. The values of the *Stress*-1 fit measure and the *HHI* index for $p = 160$ MDS procedures (with the best solution denoted by the red circle)

Source: own presentation using the mdsOpt package in R environment (R Core Team, 2023).

Table 3 contains a comparison of results for the 'best' MDS procedure (16) and the two 'worst' procedures (41 and 154).

Table 3. A comparison of results for the best and worst MDS procedures

Elements of the procedure	MDS procedure		
	16	41	154
Normalisation method	n9a	n9a	n2
Distance measure	squared Euclidean distance	GDM1	squared Euclidean distance
Model	interval	mspline of 2nd degree	ratio
	Results		
STRESS-1	0.1147	0.1346	0.2005
HHI	453.56	1068.36	970.23

Source: own presentation using the mdsOpt package in R environment (R Core Team, 2023).

Figures 2 to 4 show the results of multidimensional scaling in a two-dimensional space for the optimal procedure 16 (Figure 2) and for procedures 41 (Figure 3) and 154 (Figure 4), for which the distribution of errors (in terms of stress per point) deviated considerably from the uniform distribution. When the errors were uniformly distributed, the *HHI* index was equal to 333.33.

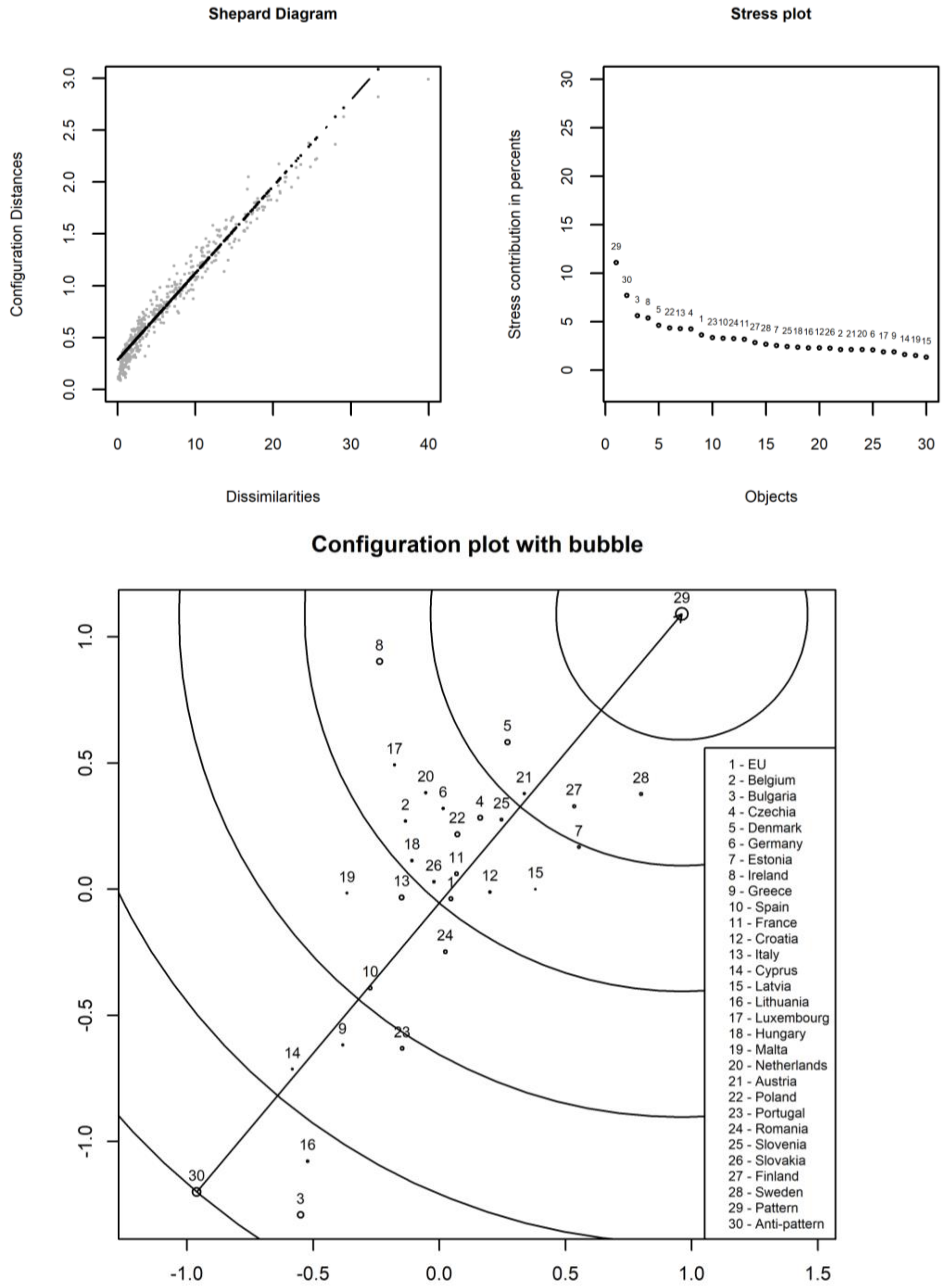

Fig. 2. The results of MDS (configuration plot with bubble) with Shepard diagram and Stress plot for procedure 16
Source: own presentation using R environment (R Core Team, 2023).

The Shepard diagram offers a detailed insight into the goodness-of-fit between distances in the multidimensional space (Dissimilarities) and the scaling space (Configuration Distances). The diagram presents a general picture of dispersion around the function of regression, making it possible to detect outliers. In the optimal solution (see the Shepard diagram in Figure 2), there are no outliers, whereas in contrast, the Shepard diagrams in Figures 3 and 4 include outliers, which are the result of the distribution of errors in the positioning of objects in a two-dimensional space (see the stress plots).

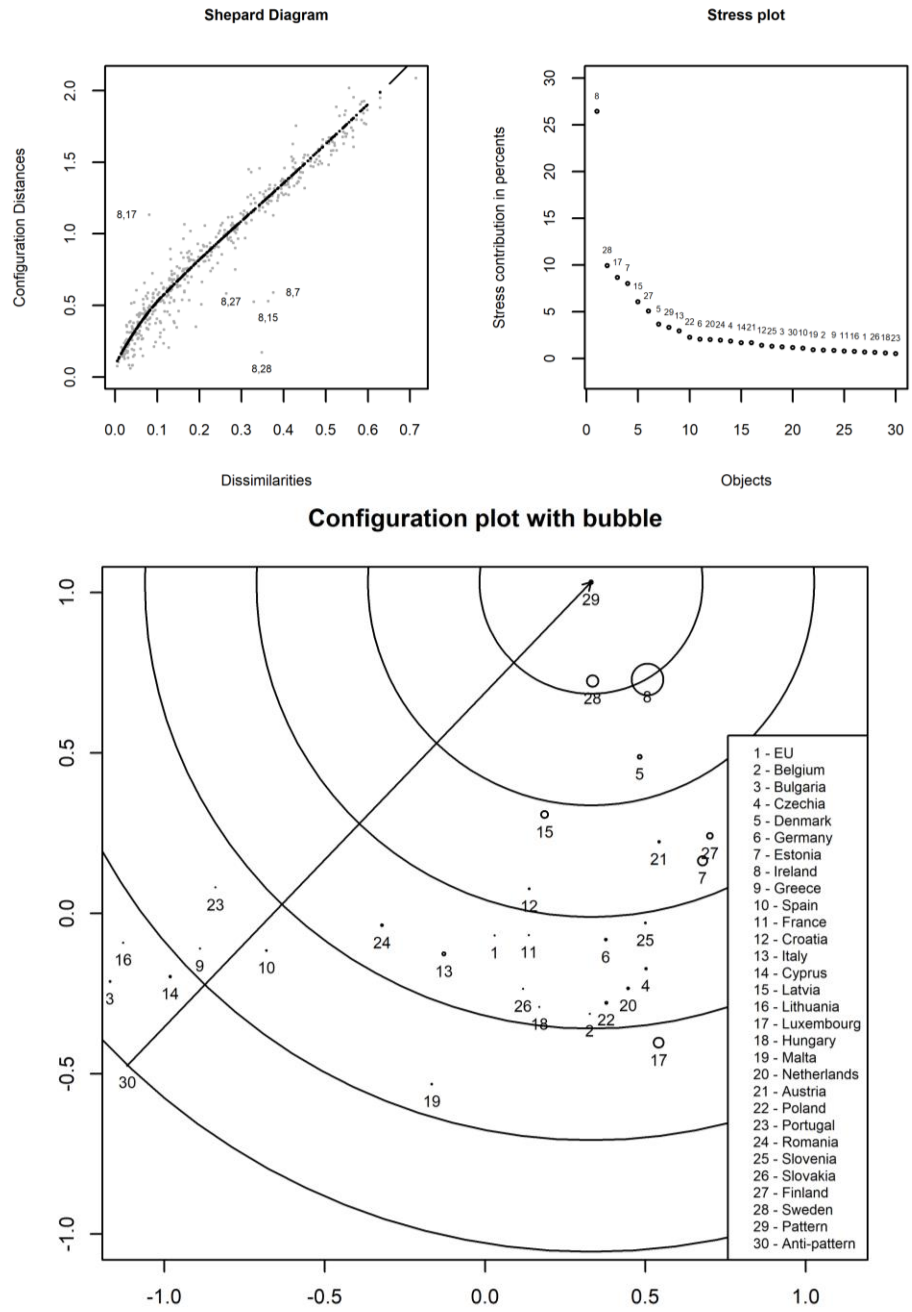

Fig. 3. The results of MDS (configuration plot with bubble) with Shepard diagram and Stress plot for procedure 41

Source: own presentation using R environment (R Core Team, 2023).

Overall stress for procedure no. 41 (0.1346) was acceptable. The stress plot in Figure 3 shows that the object representing Ireland (8) contributed a considerable share of overall stress (26.42%). Outlying objects are also clearly evident in the Shepard diagram. The MDS configuration (configuration plot with bubble) does not represent all dissimilarities equally well. With increasing values of the HHI_p index, the degree to which multidimensional scaling correctly represents the real relationships between objects' decreases.

Overall stress for procedure 154 (0.2005) was not acceptable. As evident from the stress plot in Figure 4, the pattern object (29), the anti-pattern (30) and the object representing Poland (22) made the biggest contribution to overall stress (47.51%). Moreover, the Shepard diagram shows three outliers, which disproportionally contributed to total stress. The MDS configuration (configuration plot with bubble) does not represent all the dissimilarities equally well. With increasing values of the HHI_p index, the degree to which multidimensional scaling correctly represents the real relationships between objects decreases.

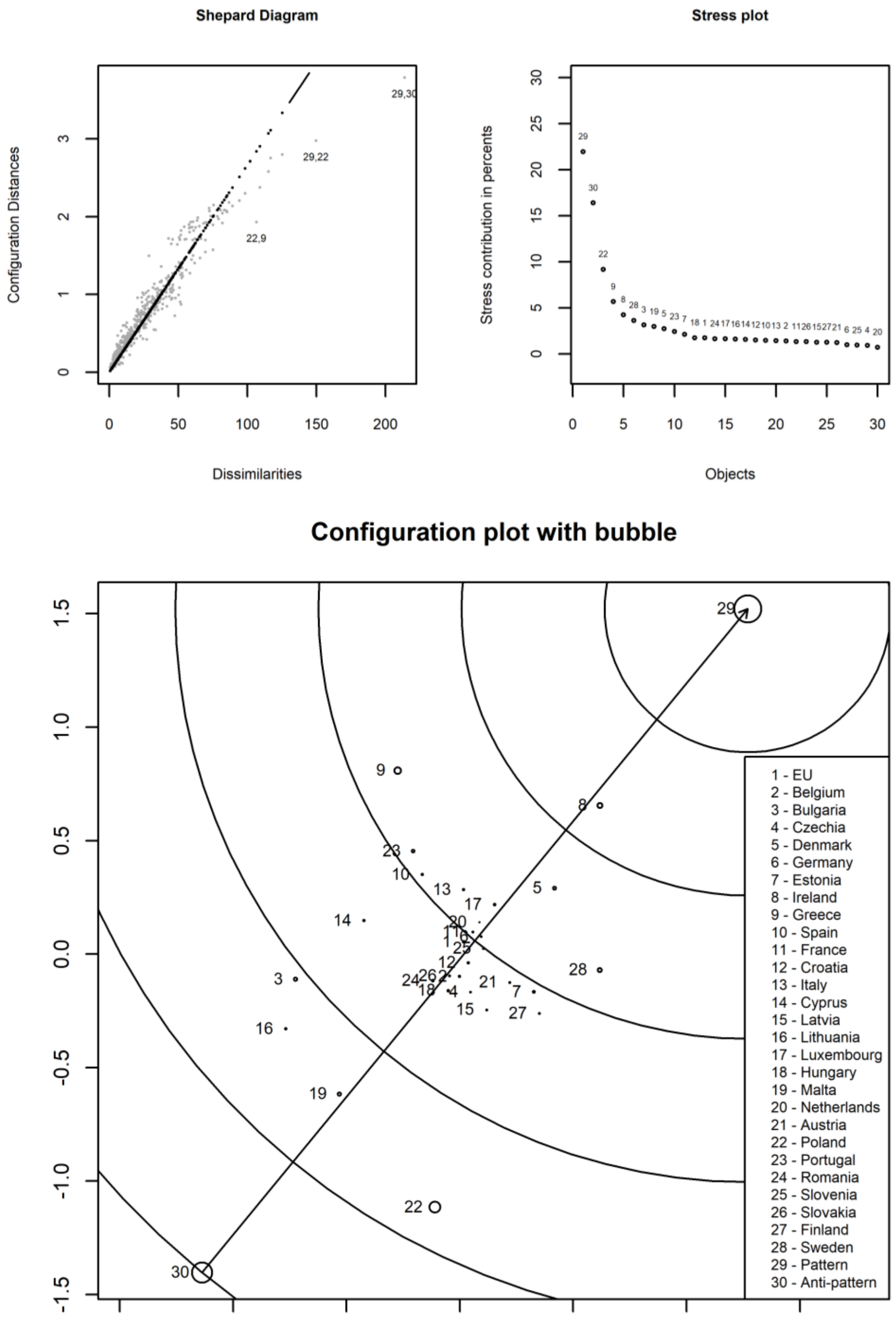

Fig. 4. The results of MDS (configuration plot with bubble) with Shepard diagram and stress plot for procedure 154

Source: own presentation using R environment (R Core Team, 2023).

The configuration of objects in a two-dimensional space for procedures 16, 41 and 154 was used as the basis for calculating the values of aggregate measure d_i according to formula (2). The ranking of EU countries in terms of their progress on the SDG7 goal in 2021 is shown in Table 4. The higher the value of d_i, the more progress a given country had made towards achieving the SDG7 goal in 2021.

Table 4. The ranking of EU countries according to their progress on the SDG7 goal in 2021 for three MDS procedures

No.	Country	Procedure 16		Procedure 41		Procedure 154	
		Distance (2)	Rank	Distance (2)	Rank	Distance (2)	Rank
28	Sweden	0.7554	1	0.8524	1	0.5459	5
5	Denmark	0.7134	2	0.7291	3	0.6047	2
27	Finland	0.7080	3	0.5814	6	0.4706	17
21	Austria	0.6834	4	0.5992	5	0.4847	15
7	Estonia	0.6623	5	0.5517	7	0.4895	14
25	Slovenia	0.6375	6	0.4847	9	0.4992	12
4	Czechia	0.6200	7	0.4171	13	0.4496	19
8	Ireland	0.5945	8	0.8320	2	0.7136	1
6	Germany	0.5921	9	0.4656	10	0.5083	10
15	Latvia	0.5869	10	0.6459	4	0.4432	21
20	Netherlands	0.5862	11	0.3908	16	0.5198	7
22	Poland	0.5832	12	0.3714	18	0.2144	28
17	Luxembourg	0.5697	13	0.3047	21	0.5469	4
12	Croatia	0.5522	14	0.5327	8	0.4751	16
11	France	0.5441	15	0.4649	11	0.5064	11
2	Belgium	0.5425	16	0.3556	20	0.4564	18
26	Slovakia	0.5162	17	0.3841	17	0.4495	20
18	Hungary	0.5154	18	0.3609	19	0.4353	22
1	EU	0.5141	19	0.4530	12	0.4907	13
13	Italy	0.4717	20	0.4028	14	0.5348	6
24	Romania	0.4536	21	0.4000	15	0.4329	23
19	Malta	0.4221	22	0.2135	24	0.2619	27
10	Spain	0.3547	23	0.2668	23	0.5109	9
23	Portugal	0.3153	24	0.2770	22	0.5193	8
9	Greece	0.2731	25	0.1998	25	0.5511	3
14	Cyprus	0.2061	26	0.1384	26	0.4243	24
16	Lithuania	0.1214	27	0.1177	27	0.2733	26
3	Bulgaria	0.0569	28	0.0661	28	0.3195	25

Source: authors' calculations using R environment (R Core Team, 2023).

Kendall's tau correlation coefficient was calculated for three procedures (16, 41 and 154) to evaluate the similarity of the rankings of EU countries in terms of their progress on the SDG7 goal in 2021. The tau value for the results of procedure 41 in relation to those resulting from procedure 16 was 0.725, and for the results of procedure 154 in relation to those obtained by applying procedure 16, it was 0.254.

When the ranking obtained by applying procedure 41 was compared with that resulting from the optimal procedure 16, one could notice a number of changes in the order of countries: Ireland moved from 8th to 2nd place, Latvia – from 10th to 4th, Luxembourg fell from 13th to 21st, Czechia – from 7th to 13th, and Poland – from 12th to 18th. The changes were due to a considerably non-uniform distribution of errors regarding the arrangement of individual objects in the scaling space, as indicated by the value of $HHI = 1068.36$ based on stress-per-point values.

When a similar comparison was made for the ranking obtained by applying procedure 154, changes in the order of countries were even more evident: Ireland moved up from 8th to 1st place, Luxembourg – from 13th to 4th, Finland dropped from 3rd to 17th, and Poland – from 12th to 28th. The resulting ranking is largely random because of a very poor fit of the MDS configuration, as indicated by the value of $Stress-1 = 0.2005$ and a considerably non-uniform distribution of errors regarding the arrangement of individual objects in the scaling space, as measured by the value of $HHI = 970.23$.

4. A list of studies involving the hybrid method

Table 5 presents an overview of studies involving the hybrid method, which combines linear ordering with multidimensional scaling, together with information about data type, approach (static or dynamic), data source and the research problem.

Table 5. An overview of studies involving the hybrid method, which combines linear ordering with multidimensional scaling

No.	Article	Data type	Approach	Data source	Research problem
1	Walesiak (2016)	Metric	Static	Secondary Statistics Poland	An assessment of tourist attractiveness of 29 districts of Dolnośląskie Province in 2014
2	Walesiak (2017a)	Metric	Dynamic	Secondary Statistics Poland	Measuring and assessing changes in the level of social cohesion across districts of Dolnośląskie Province in 2005-2015
3	Walesiak (2017b)	Ordinal	Static	Primary	Linear ordering of 27 properties described by 6 ordinal variables in the real estate market of Jelenia Góra in terms of their attractiveness
4	Walesiak, & Dudek (2017)	Metric	Static	Secondary Statistics Poland	An assessment of tourist attractiveness of 29 districts of Dolnośląskie Province in 2014
5	Dehnel et al. (2018)	Metric	Dynamic	Secondary Eurostat	An assessment of changes regarding population aging in 35 regions of the Visegrad Group in 2016 compared with 2005
6	Walesiak, & Dehnel (2018)	Interval-valued	Static	Secondary Statistics Poland	An assessment of economic efficiency of small manufacturing companies in districts of Wielkopolskie Province
7	Dehnel, & Walesiak (2019)	Metric, Interval-valued	Static	Secondary Statistics Poland	An assessment of economic efficiency of small manufacturing companies in districts of Wielkopolskie Province
8	Dehnel et al. (2019)	Metric, Interval-valued	Static	Secondary Statistics Poland	A comparative analysis of rankings of Poland's provinces in terms of social cohesion in 2016
9	Obrębalski, & Walesiak (2019)	Metric	Dynamic	Secondary Eurostat	Measuring the degree of variation in the labour market situation of people aged 15-24 in border areas of Poland, Czechia, and Germany in 2010 and 2018 using 6 variables
10	Walesiak, & Dehnel (2019b)	Metric	Dynamic	Secondary Statistics Poland	A comparison of the degree of population aging across Poland's provinces in 2002, 2010 and 2017 in terms of median age, old age rate, double ageing index, ageing index, old-age dependency ratio
11	Walesiak, & Dehnel (2019a)	Metric, Interval-valued	Static	Secondary Statistics Poland	A comparative analysis of rankings of Poland's provinces in terms of social cohesion in 2016
12	Walesiak, & Dehnel (2020)	Metric, Interval-valued	Static	Secondary Statistics Poland	A comparative analysis of rankings of Poland's provinces in terms of social cohesion in 2018
13	Dehnel et al. (2020)	Metric	Dynamic	Secondary Eurostat	An assessment of changes regarding population aging in regions of the Visegrad Group in 2016 compared with 2005
14	Walesiak, & Dudek (2020)	Metric, Interval-valued	Static	Secondary Statistics Poland	An assessment of tourist attractiveness of districts of Dolnośląskie Province using metric data An assessment of tourist attractiveness of Poland's provinces using interval-valued data

Source: authors' compilation.

The information in Table 5 can be summarised as follows:

- data type: 12 studies based on metric data (the starting point was a data matrix or a data cube); in 6 studies based on interval-valued data (the starting point is a data table) and one based on ordinal data (the starting point was a data matrix),
- approach: 9 studies involving the static approach and 5 – the dynamic approach,
- data source: 13 studies based on secondary data from Statistics Poland (10) and from Eurostat (3), and just 1 was based on primary data.

5. Conclusions

The article offers an overview of the methodological solutions regarding the hybrid method, which combines multidimensional scaling and linear ordering, taking into account different types of data (metric, ordinal, interval-valued). The purpose of the review was to assess the applicability of the various methods taking into consideration the insights they can provide as well as the problems associated with the use of multidimensional scaling for the purpose of linear ordering. Multidimensional scaling can be used to visualise the results of linearly ordered objects in a two-dimensional space. The graphic presentation was enriched by the inclusion of development isoquants (curves of equal development) and a development path (i.e. the shortest line connecting the development pattern and anti-pattern). It was not possible to visualise the results in this way using other linear ordering methods mentioned in Section 1.1.

The problems encountered when applying multidimensional scaling in linear ordering are exemplified by the results of the study in which 27 EU countries were ranked according to their progress towards reaching the sustainable development goal (SDG7) in 2021. When the distribution of errors regarding the arrangement of objects (countries) in the scaling space, measured by values of stress-per-point, deviated considerably from the uniform distribution, the resulting arrangement of objects in a two-dimensional space was incorrect. The problem could be solved by selecting an optimal multidimensional scaling procedure, taking into account various normalisation methods, distance measures, and scaling models (ratio, interval, mspline). Two criteria were applied to select the optimal MDS procedure: Kruskal's $Stress$-1, which is a measure of fit (standardised residual sum of squares), and the Herfindahl-Hirschman HHI index, calculated on the basis of stress-per-point values.

The article contains a list of studies involving the hybrid method, including information about the type of data used in the analysis (metric, ordinal, interval-valued), the approach (static or dynamic), data source (primary or secondary) and the research problem.

The use of the hybrid method was facilitated by the functions implemented in the mdsOpt package in R environment (Walesiak, & Dudek 2023b).

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Received: January 2024, revised: September 2024