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The use of transitive Montgomery indicators for scanner data analysis

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Abstract: Although the modern price index theory is based on an analysis of ratios of prices and quantities, one may be often more interested in working with differences in these values in many economic areas, e.g.: revenue change decompositions, profit and cost change decompositions, or an analysis of changes in consumer surplus. The benefit of using these differences is that there is no problem associated with the occurrence of zero prices and quantities, a problem that arises when working with ratios. In practice, one mostly cares about decomposing the value difference into indicators of contributions from price and quantity differences. The well-known price and quantity indicators are the Bennet and the Montgomery indicators, which are not transitive. This paper revises the price and quantity Montgomery indicators and their multilateral versions for the analysis of scanner data. Specifically, instead of considering 'classical' comparisons across firms, countries or regions, the transitive versions of the Montgomery indicators were adapted to work on scanner data sets observed over a fixed time window. One of the objectives of the study was to compare bilateral and multilateral Montgomery indicator values for different data aggregation levels and three main types of data filters. To the best of the authors' knowledge, this study is pioneering on the grounds of implementing the multilateral Montgomery indicators in scanner data analysis.

Keywords: scanner data, Montgomery indicators, transitivity, multilateral indicators

1. Introduction

The contemporary price index theory is based on comparisons of ratios of prices, quantities and expenditures of goods and services (von der Lippe, 2007; International Labour Office, 2004; International Monetary Fund, 2020). These index numbers are used to build various economic measures, such as gross domestic product (GDP), producer price index (PPI) and consumer price index (CPI). Nevertheless, in many business contexts one may be more interested in the magnitude of differences in prices, quantities and sale values. This may concern many economic areas, including revenue change decompositions, profit and cost change decompositions, or the analysis of changes in consumer surplus (Diewert, 2005). An important benefit of using such differences is that there is no problem associated with the occurrence of zero prices and quantities, a problem that arises when working with ratios. The issue of zero prices or zero quantities may be very serious in many business applications where not all goods are produced and purchased in every period.

Zero prices and quantities are also common in scanner data sets due to the high product churn observed in supermarkets. Scanner data, which support CPI calculations in many countries, mean transaction data that specify turnover and numbers of items sold by barcodes, e.g. Global Trade Item Number (GTIN), European Article Number (EAN), and Stock Keeping Unit (SKU) codes. Scanner data have numerous advantages compared to traditional survey data collection because such data sets are much bigger than traditional ones and they contain complete transaction information, namely information about prices and quantities at barcode level and also many other product characteristics (i.e. product weight sales unit, product label, VAT level, etc.). Nevertheless, at barcode level, the occurrence of zero-scanner prices is quite common because supermarkets often offer a variety of discounts and price reductions (even down to zero), as well as a significant turnover of products. This causes analytical problems both for statistical offices and for supermarket owners wishing to compare the sales performance of different product segments in two time periods. Thus, an approach based on the differences in price values, quantities and expenditure can also be very useful in analysing scanner data.

The difference-based approach to index numbers is well established in the economic literature, where it was introduced in the early 20th century (Bennet, 1920; Montgomery, 1929). Note that index numbers, expressed in terms of differences, are referred to as indicators (Diewert, 2005). Recently, one can witness a return of interest in this approach on the part of statisticians and economists (Balk *et al*., 2004; Diewert, 2005; Fox, 2006; Cross and Färe, 2009; de Boer and Rodrigues, 2020). However, to the best of the authors' knowledge, there is a lack of studies that apply the indicators to the analysis of scanner data, which is the main objective of this paper focused on the Montgomery indicators because they receive much less attention in the literature than the Bennet indicators.

The added value of the paper and its contribution to the existing knowledge about indicators can be summarised as follows: (1) the Montgomery indicators and their transitive versions defined for comparisons across companies (or regions) in cross-section or panel context are adopted to work on scanner data sets observed over a fixed time window; (2) the axiomatic properties of the Montgomery multilateral indicators are verified; (3) variants of the Montgomery indicators based on matched samples are considered, which may be more accurate due to the high turnover of scanner products; (4) the impact of data filters used and the level of data aggregation on the price and quantity Montgomery indicators is also examined.

The structure of the paper is as follows: Section 2 presents the bilateral Montgomery indicators, Section 3 adopts the transitive Montgomery indicators from the field of comparisons between companies and examines their axiomatic properties, Section 4 presents the results of the empirical study in which the bilateral and multilateral Montgomery indicators were applied to the analysis of scanner data, and Section 5 lists the most important conclusions drawn from the empirical study.

2. The Montgomery indicators

The axiomatic and economic index theory is derived from Fisher (1922) and Konűs (1939), respectively. Although until the end of the 20th century there was a lack of studies providing a foundation for the construction of price and quantity indices, more recently there has been a marked interest among statisticians and economists in the indicators theory. For instance, Chambers (2001) proposed a new economic framework for indicators by using Diewert's (1976) quadratic lemma. Balk *et al*. (2004) developed the theory of economic price and quantity indicators by deriving an exact relation between indicators and directional distance functions. Finally, in the paper by Diewert (2005), an additive test approach was developed.

As a rule, price and quantity indicators are calculated using firm-level price and quantity data. Authors use the context of the production theory or concentrate on the input side of companies or regions (Balk *et al*., 2004; Cross and Färe, 2009; Fox, 2006). To the best of the authors' knowledge, this study is pioneering in the area of implementing the multilateral Montgomery indicators in scanner data analysis. In this section, the bilateral Montgomery indicator formula is expressed with the additional distinction between available and matched products. The above-mentioned distinction does matter in the case of scanner data due to the high product churn. Section 3 goes a step further by proposing a multilateral approach, where the transitive version of the Montgomery indicators takes the entire given time window into account.

2.1. The case of a matched product sample

Due to the high turnover of scanner products, a frequently used approach in determining price indices is the matched sample approach. In order to correctly define the Montgomery indicators in the matched sample approach, first enter the necessary notation.

Denote sets of homogeneous products which belong to the same product groups (category) in months 0 and t by G_0 and G_t respectively, with $G_{0,t}$ denoting a set of matched products in both considered periods, i.e. $G_{0,t} = G_0 \cap G_t$. Let p_i^τ and q_i^τ denote the price (more precisely: *unit value*) and quantity of product *i* at time $\tau \in \{0, t\}$, which are assumed to be positive. Note that the *Vartia mean* (Vartia, 1976) of two positive real numbers *x* and *y*, which is known in the mathematical literature as the *logarithmic mean* (von der Lippe, 2007), is defined as below.

$$L(x,y) = \begin{cases} \frac{x-y}{\ln(x)-\ln(y)}; & x \neq y, \\ x; & x = y. \end{cases} \tag{1}$$

Under the above, the bilateral Montgomery price indicators, which compare current period *t* to base period 0 and consider only the matched products, can be defined as follows:

$$_mIP_{0,t}^M = \sum_{i\in G_{0,t}} L(p_i^t q_i^t, p_i^0 q_i^0) ln\left(\frac{p_i^t}{p_i^0}\right), \tag{2}$$

and the bilateral Montgomery quantity indicator can be expressed as follows:

$$_mIQ_{0,t}^M = \sum_{i\in G_{0,t}} L(p_i^t q_i^t, p_i^0 q_i^0) ln\left(\frac{q_i^t}{q_i^0}\right). \tag{3}$$

It is easy to show (Montgomery, 1929) that the consumer's value change over the two periods under consideration, observed for the matched products, can be decomposed by using the Montgomery price and quantity indicators, i.e.

$$V_{G_{0,t}}^t - V_{G_{0,t}}^0 \equiv \sum_{i\in G_{0,t}} p_i^t q_i^t - \sum_{i\in G_{0,t}} p_i^0 q_i^0 = {}_mIP_{0,t}^M + {}_mIQ_{0,t}^M. \tag{4}$$

2.2. The case of unmatched products

Note that in the case of scanner data, the set of products available in the base and current periods, i.e. $G_0^t = G_0 \cup G_t$, is generally an oversampling of the set of matched products $G_{0,t} = G_0 \cap G_t$. Regarding the Bennet indicators, this does not generate technical problems in their calculation since it is simply assumed that a product that is available in period t_1, but is not available in period t_2, has zero price and quantity in period t_2. Such a procedure does not apply to the Montgomery indicators since they use logarithms of prices and quantities in their syntax. However, it can be shown (see Appendix 1) that prices and quantities of mismatched products in the periods being compared can effectively be determined in line with the following procedure:

$$p_i^t = q_i^t = \varepsilon, \text{ for } i \in G_0 \backslash G_t, \text{ and } p_i^0 = q_i^0 = \varepsilon, \text{ for } i \in G_t \backslash G_0, \tag{5}$$

which holds for a sufficiently small, real number $\varepsilon > 0$.

With the above-presented procedure for treating mismatched products, analogously to Section 2.1, one can define the Montgomery indicators for all available products (including mismatched products) as follows:

$$IP_{0,t}^M = \sum_{i \in G_0^t} L(p_i^t q_i^t, p_i^0 q_i^0) ln\left(\frac{p_i^t}{p_i^0}\right), \tag{6}$$

$$IQ_{0,t}^M = \sum_{i \in G_0^t} L(p_i^t q_i^t, p_i^0 q_i^0) ln\left(\frac{q_i^t}{q_i^0}\right). \tag{7}$$

Note that in general, when $G_0^t \neq G_{0,t}$, the Montgomery indicators (6) and (7) are functions of parameter ε (under assumption (5)), and thus they can be denoted by $IP_{0,t}^M(\varepsilon)$ and $IQ_{0,t}^M(\varepsilon)$. The observation in Appendix 1 leads to the conclusion that the Montgomery indicators satisfy the *sum test* (otherwise known as the *value change test*) with a good approximation if procedure (5) is taken into consideration, i.e.

$$V_{G_0^t}^t - V_{G_0^t}^0 \equiv \sum_{i \in G_t \backslash G_0} p_i^t q_i^t - \sum_{i \in G_0 \backslash G_t} p_i^0 q_i^0 + \sum_{i \in G_{0,t}} p_i^t q_i^t - \sum_{i \in G_{0,t}} p_i^0 q_i^0 = \lim_{\varepsilon \to 0^+} (IP_{0,t}^M(\varepsilon) + IQ_{0,t}^M(\varepsilon)). \tag{8}$$

Remark 1. In the empirical study (see Section 4), the *PriceIndices* package (Bialek, 2022) is used, in which the implementation of the Montgomery indicators takes $\varepsilon = 0.000001$. Such a small value of parameter ε means that the *sum test* is approximately satisfied in the case of these indicators. However, it should be noted that for the static item universe, i.e. when $G_0^t = G_{0,t}$, the *sum test* is fully satisfied and no proxy is needed (Diewert and Mizobuchi, 2009). One should also keep in mind that the Montgomery indicators satisfy most of the required tests except for the *monotonicity tests* and some *symmetry tests*. Those interested in the mathematical properties of the Montgomery indicators are referred to Diewert (2005).

3. The multilateral Montgomery indicators

For international or inter-regional comparisons, *transitivity* means that estimates of price dynamics and quantities of selected attributes do not depend on the choice of the underlying country or region. Similarly, for comparisons across companies, computing transitive price and quantity indices (indicators) do not depend on the choice of the company benchmark. A lack of index transitivity is a well-known problem in the literature on international comparisons and scanner data (Gini, 1931; Eltetö and Köves, 1964; Szulc, 1964; Ivancic *et al.*, 2011; Chessa, 2016). By definition, *transitivity* eliminates the chain drift problem. The chain drift can be formalised in terms of the violation of the *multi-period identity test*, according to which one can expect that when all prices and quantities in a current period revert back to their values from the base period, the index should indicate no price change and equal one. In the same situation, the price indicator should equal zero. Even chain superlative indices may suffer from chain drift (Białek and Roszko-Wójtowicz, 2023). Multilateral price indices are *transitive*, which means that the calculation of the price dynamics for any two moments in the time window does not depend on the choice of the base period (Chessa, 2016). Thus, this section focuses on forming the multilateral Montgomery indicators.

In the case of any price indicator IP and quantity indicator IQ, *transitivity* means that the following relations occur for any $0 < s < t$:

$$IP_{0,s} + IP_{s,t} = IP_{0,t} \text{ and } IQ_{0,s} + IQ_{s,t} = IQ_{0,t}. \tag{9}$$

It is easy to verify that bilateral Montgomery indicators are not transitive. In fact, using the *coffee* data set from the *PriceIndices* R package (Białek, 2022), taking *January 2019* as base period 0, *February 2019* as internal period *s*, and *March 2019* as base period *t*, and running the *Montgomery()* function, one obtains: $IP_{0,s}^{M} + IP_{s,t}^{M} =$ -49493.63 $\neq IP_{0,t}^{M} =$ -26389.21. The following design of the multilateral Montgomery indicators is an adaptation of Fox's (2006) transformation for the scanner data case, considering some time windows with more than two periods instead of some number of companies or regions.

Let us denote by [0,*T*] the considered time interval, which typically (while computing multilateral indices) consists of 13 or 25 months (Eurostat, 2022), while $G_{[0,T]}$ denotes the set of available products in the whole interval [0,*T*], i.e. $G_{[0,T]} = \bigcup_{\tau=0}^{T} G_{\tau}$, and let $G_{[0,T]}^{m}$ denote the set of matched products in the whole interval [0,*T*], i.e. $G_{[0,T]}^{m} = \bigcap_{\tau=0}^{T} G_{\tau}$. Thus one can introduce the additional notations:

$${}_{T}IP_{\tau,t}^{M} = \sum_{i \in G_{[0,T]}} L\left(p_i^t q_i^t, p_i^{\tau} q_i^{\tau}\right) ln\left(\frac{p_i^t}{p_i^{\tau}}\right), \tag{10}$$

$${}_{mT}IP_{\tau,t}^{M} = \sum_{i \in G_{[0,T]}^{m}} L\left(p_i^t q_i^t, p_i^{\tau} q_i^{\tau}\right) ln\left(\frac{p_i^t}{p_i^{\tau}}\right), \tag{11}$$

$${}_{T}IQ_{\tau,t}^{M} = \sum_{i \in G_{[0,T]}} L\left(p_i^t q_i^t, p_i^{\tau} q_i^{\tau}\right) ln\left(\frac{q_i^t}{q_i^{\tau}}\right), \tag{12}$$

$${}_{mT}IQ_{\tau,t}^{M} = \sum_{i \in G_{[0,T]}^{m}} L\left(p_i^t q_i^t, p_i^{\tau} q_i^{\tau}\right) ln\left(\frac{q_i^t}{q_i^{\tau}}\right). \tag{13}$$

It should be noted that the previously assumed procedure (5) still applies to formulas (10) and (12). Defining a multilateral Montgomery indicator for all available products from a time window requires first introducing the following averaged price and quantity indicators:

$$I\bar{P}_{t_0}^{M} = \frac{1}{T+1} \sum_{\tau=0}^{T} {}_{T}IP_{\tau,t_0}^{M}, \tag{14}$$

$$I\bar{Q}_{t_0}^{M} = \frac{1}{T+1} \sum_{\tau=0}^{T} {}_{T}IP_{\tau,t_0}^{M}. \tag{15}$$

Now, using (14) and (15), the price and quantity multilateral Montgomery indicator can be defined respectively:

$$MIP_{0,t}^{M} = I\bar{P}_{t}^{M} - I\bar{P}_{0}^{M}, \tag{16}$$

$$MIQ_{0,t}^{M} = I\bar{Q}_{t}^{M} - I\bar{Q}_{0}^{M}. \tag{17}$$

Note that the multilateral Montgomery indicators (16) and (17) are *transitive*, i.e.

$$MIP_{0,s}^{M} + MIP_{s,t}^{M} = I\bar{P}_{s}^{M} - I\bar{P}_{0}^{M} + I\bar{P}_{t}^{M} - I\bar{P}_{s}^{M} = I\bar{P}_{t}^{M} - I\bar{P}_{0}^{M} = MIP_{0,t}^{M}, \tag{18}$$

$$MIQ_{0,s}^{M} + MIQ_{s,t}^{M} = I\bar{Q}_{s}^{M} - I\bar{Q}_{0}^{M} + I\bar{Q}_{t}^{M} - I\bar{Q}_{s}^{M} = I\bar{Q}_{t}^{M} - I\bar{Q}_{0}^{M} = MIQ_{0,t}^{M}. \tag{19}$$

These multilateral Montgomery indicators satisfy the *time reversal test* since (20) and (21) hold:

$$MIP_{0,t}^{M} = I\bar{P}_{t}^{M} - I\bar{P}_{0}^{M} = -(I\bar{P}_{0}^{M} - I\bar{P}_{t}^{M}) = -MIP_{t,0}^{M}, \tag{20}$$

$$MIQ_{0,t}^{M} = I\bar{Q}_{t}^{M} - I\bar{Q}_{0}^{M} = -(I\bar{Q}_{0}^{M} - I\bar{Q}_{t}^{M}) = -MIQ_{t,0}^{M}. \tag{21}$$

It can be proven that the multilateral Montgomery indicators also satisfy the *value change test* and the *multi-period identity test* (see Appendix 2).

Remark 2. Let us conclude this section by pointing out that all the axiomatic properties valid for the multilateral Montgomery indicators calculated on the set of all available products $G_{[0,T]}$ also carry over to the analogous multilateral Montgomery indicators estimated on the set of matched products $G_{[0,T]}^{m}$. The transitive Montgomery indicators based on matched products can be written as

$$_{m}MIP_{0,t}^{M} = \frac{1}{T+1}\sum_{\tau=0}^{T}\left({}_{mT}IP_{\tau,t}^{M} - {}_{mT}IP_{\tau,0}^{M}\right), \tag{22}$$

$$_{m}MIQ_{0,t}^{M} = \frac{1}{T+1}\sum_{\tau=0}^{T}\left({}_{mT}IQ_{\tau,t}^{M} - {}_{mT}IQ_{\tau,0}^{M}\right), \tag{23}$$

where, similarly to the proof from **Appendix 2**, it can be proven that

$$_{m}MIP_{0,t}^{M} + {}_{m}MIQ_{0,t}^{M} = V_{G_{[0,T]}^{m}}^{t} - V_{G_{[0,T]}^{m}}^{0}. \tag{24}$$

4. Empirical study

This empirical study utilised scanner data obtained from a single retail chain operating in Poland. Specifically, the authors analysed monthly data pertaining to *stationery and hygiene products* (COICOP 5: 121322), taken from December 2021 to December 2022 and include more than 500 outlets. The COICOP 5 product group consists of the following local COICOP 6 product subgroups: tissues (60 products, i.e. 66 product identifiers (IDs)), wet wipes (88 IDs), toilet paper (117 IDs), baby diapers (193 IDs), sanitary pads (20 IDs), sanitary napkins (67 IDs), and tampons (22 IDs).

To proceed with the calculations, the data had to be properly prepared, and functions from the *PriceIndices* R package (Białek, 2021) were used for this purpose. One of the first functions was *data selecting()*, which required the creation of dictionaries containing the relevant keywords and phrases that were necessary to identify distinct product groups. The second function – *data_classification()* was utilised to handle problematic products that were previously unclassified, necessitating the manual preparation of learning samples using historical data. Next, product matching was carried out by leveraging available Global Trade Item Number (GTIN) barcodes, internal retail chain codes, and product labels. The *PriceIndices* package facilitated this task through the use of the *data_matching()* function.

Different variants were considered for comparing the sales value difference: the Montgomery price indicator (referred to as the “price effect” in the figure) and the Montgomery quantity indicator (referred to as the “quantity effect”). Calculations were performed to analyse and assess these indicators across the various variants.

Figures 1 and 2 show the bilateral versions of the Montgomery indicators, while Figures 3 and 4 present the multilateral versions which are equivalent to the first two figures. The bilateral indicators, similar to the multilateral indicators, were examined in two scenarios: one without filtering the original data (as depicted in Figures 1 and 3), and the other with the implementation of a low sales filter using parameter λ = 1.25 (Loon and Roels, 2018).

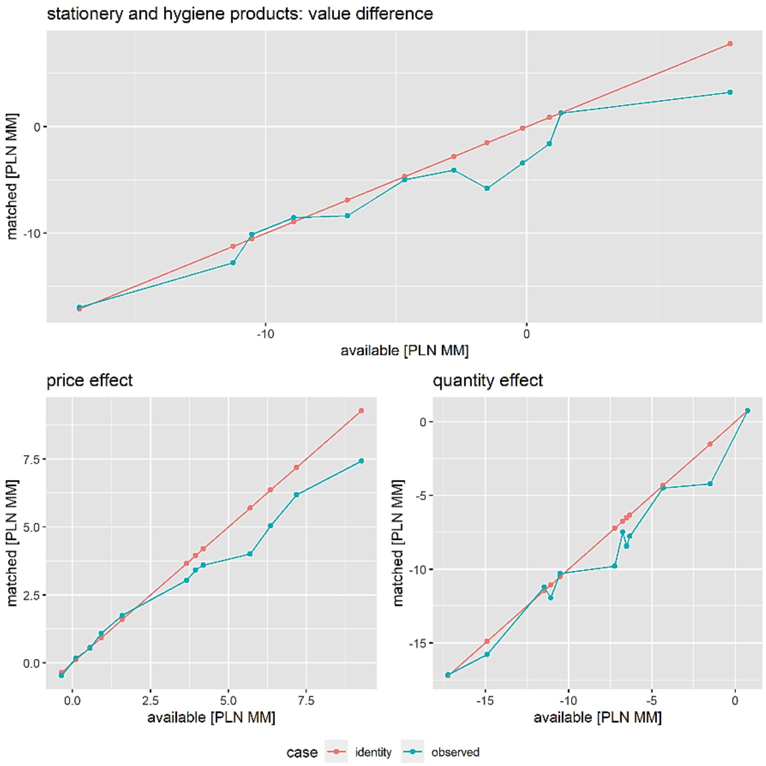

Fig. 1. Comparison of the difference in the value of sales and the bilateral Montgomery indicators calculated for all available products and for matched products (no data filters applied)

Source: own calculations in the *PriceIndices* R package.

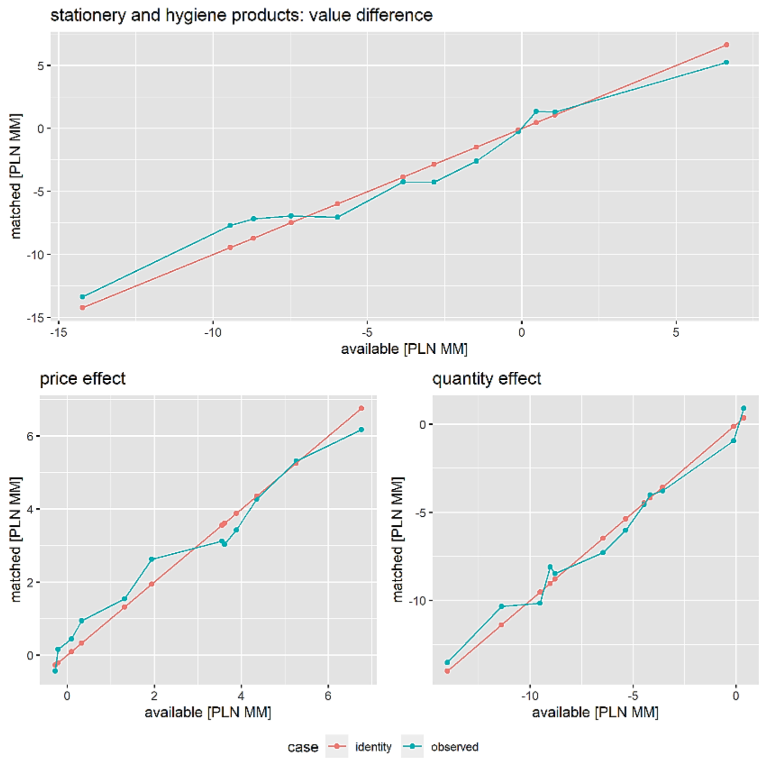

Fig. 2. Comparison of the difference in the value of sales and the bilateral Montgomery indicators calculated for all available products and for matched products (data filters applied)

Source: own calculations in the *PriceIndices* R package.

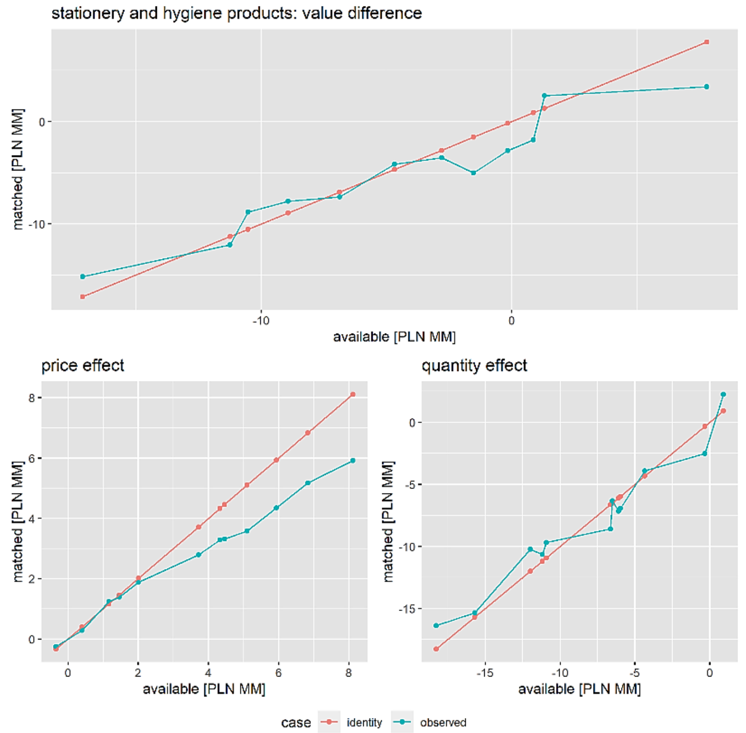

Fig. 3. Comparison of the difference in the value of sales and the multilateral Montgomery indicators calculated for all available products and for matched products (no data filters applied)

Source: own calculations in the *PriceIndices* R package.

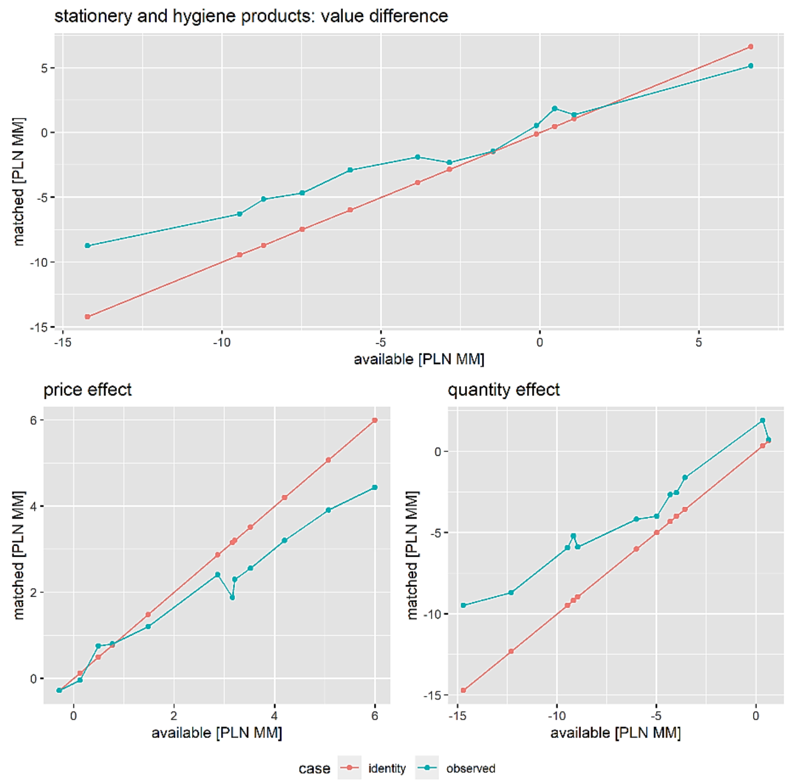

Fig. 4. Comparison of the difference in the value of sales and the multilateral Montgomery indicators calculated for all available products and for matched products (data filters applied)

Source: own calculations in the *PriceIndices* R package.

On the OX axis, the values calculated in ascending order for all months within the analysed time window, representing the characteristics under study (value difference, price or quantity effect), were plotted for all available products within that time frame. On the OY axis, similar values for the corresponding months were indicated, but this time only the products that match the criteria were included.

The presented figures can be interpreted as follows: if the green line (called “empirical”) is below the red line (called “identity”), this means that product matching caused a decrease in the values of the characteristics under study. The greater the deviation between the empirical line and the theoretical one, the stronger the effect mentioned above.

In the bilateral approach, the impact of data filtering appeared to go in the opposite direction than in the multilateral approach. For the bilateral approach, one can see that data filtering decreases the product matching effect (Figures 1 and 2). At the same time, data filtering increases the product matching effect in the multilateral approach (Figures 3 and 4). In other words, for the Montgomery’s bilateral indicators, after filtering, smaller differences were observed between the results of price and quantity indicators calculated for matched and available products. In contrast, the differences were greater when filtering was applied to multilateral indicators (see Figure 4). In this case, the red and green lines in the figure were clearly separated, indicating that the matched and unmatched approaches generated markedly different scores for price and quantity indicators and, consequently, for the total value change. It can be further noted that the price effect was weakened after filtering in the multilateral approach when only matched products were included, but simultaneously the quantity effect was strengthened.

To be more specific, the Montgomery indicators’ values (in all the versions) were calculated for the product precisely defined at two levels: the narrow level using the GTIN code and the broader level using COICOP 6 classification. For a better and more accurate view, a series of comparisons were made for the current period set at the end of the time window (December 2022). Their results are shown in Tables 1 and 2.

Table 1. Comparison of the Montgomery indicators across the data aggregation level and data filtering (all the available products were considered, the normalised values are in brackets)

GTIN level: bilateral approach				
Characteristics	No data filtering		With data filtering	
Sales value difference	7769806.28	(100)	6624556.59	(100)
Price Montgomery indicator	9275020.81	(119.37)	6755626.25	(101.98)
Quantity Montgomery indicator	-1505214.53	(-19.37)	-131069.66	(-1.98)
GTIN level: multilateral approach				
Sales value difference	7769806.28	(100)	6624556.59	(100)
Price Montgomery indicator	8110048.49	(104.38)	5990950.99	(90.44)
Quantity Montgomery indicator	-340242.21	(-4.38)	633605.60	(9.56)
COICOP 6 level: bilateral approach				
Characteristics	No data filtering		With data filtering	
Sales value difference	7769806.28	(100)	6624556.59	(100)
Price Montgomery indicator	9895893.58	(127.36)	8018053.51	(121.04)
Quantity Montgomery indicator	2126087.30	(-27.36)	-1393496.92	(-21.04)
COICOP 6 level: multilateral approach				
Characteristics	No data filtering		With data filtering	
Sales value difference	7769806.28	(100)	6624556.59	(100)
Price Montgomery indicator	8298307.45	(106.8)	6743673.56	(101.8)
Quantity Montgomery indicator	-528501.17	(-6.8)	-119116.97	(-1.8)

Source: own calculations in the *PriceIndices* R package.

Table 2. Price and quantity contributions across the bilateral and multilateral approach (all the available products were considered, the normalised values are in brackets: total sales value difference = 100)

Product contributions: bilateral approach						
COICOP 6 subgroup	Sales value difference		Price contributions		Quantity contributions	
Tissues	2423488.86	(36.58)	1745602.68	(26.35)	677886.18	(10.23)
Toilet paper	-149921.79	(-2.26)	91729.22	(1.38)	-241651.01	(-3.65)
Baby diapers	3200407.98	(48.31)	5897172.95	(89.02)	-2696764.97	(-40.71)
Sanitary pads	1142085.37	(17.24)	207101.43	(3.13)	934983.94	(14.11)
Sanitary napkins	112069.03	(1.69)	45314.16	(0.68)	66754.87	(1.01)
Tampons	-103572.86	(-1.56)	31133.07	(0.47)	-134705.93	(-2.03)
Product contributions: multilateral approach						
COICOP 6 subgroup	Sales value difference		Price contributions		Quantity contributions	
Tissues	2423488.86	(36.58)	1412934.66	(21.33)	1010554.20	(15.25)
Toilet paper	-149921.79	(-2.26)	71219.97	(1.08)	-221141.76	(-3.34)
Baby diapers	3200407.98	(48.31)	4963225.03	(74.92)	-1762817.05	(-26.61)
Sanitary pads	1142085.37	(17.24)	220984.24	(3.34)	921101.13	(13.90)
Sanitary napkins	112069.03	(1.69)	50149.98	(0.76)	61919.05	(0.93)
Tampons	-103572.86	(-1.56)	25159.68	(0.38)	-128732.54	(-1.94)

Source: own calculations in the *PriceIndices* R package.

When calculating the Montgomery indicators for data specified with GTIN accuracy, it can be seen that the absolute values of the bilateral indicators are greater than the absolute values of the multilateral ones when the data are not filtered (Table 1). For filtered data, the price and quantity effects are accentuated when switching from the bilateral to multilateral approaches. A slightly different conclusion applied to the data aggregated at COICOP 6 level. In this case, in both the unfiltered and filtered data variants, the transition to the multilateral indicators had flattened assessments of the price and quantity effect, i.e. the absolute values of price and quantity multilateral indicators were smaller. Since multilateral indicators take into account a much wider range of data, this may mean that bilateral indicators overestimate the price and quantity effect, especially for unfiltered data.

At the same time, the normalised shares of individual COICOP 6 product subgroups in creating the price and quantity effect appeared to be similar if one compared the bilateral and multilateral approaches (see Table 2). Importantly, for each of the six product subgroups, the sign for the assessment of the price and quantity effect did not change when switching from the bilateral approach to the multilateral one, although the nominal values (non-normalised) were radically different.

5. Conclusions

The adaptation of the Montgomery transitive indicators to multilateral versions, operating on a fixed time window, appears to be a valuable addition to the analysis of scanner data due to the product churn that occurs here as a rule. A valuable result obtained from the study was the conclusion that the bilateral and multilateral Montgomery indicators differ not only in terms of tests (axioms) they satisfy, but also due to the fact that these indicators generate different values regardless of the level of data aggregation. As shown in Appendix 1, the use of the Montgomery indicators is also possible when considering all products available in the periods being compared by using a simple procedure of replacing zero prices and quantities with sufficiently small numbers. To the best of the authors' knowledge, this article constitutes the first application of the Bennet multilateral indicators (in "matched" and "available" versions) in the analysis of scanner data.

The main practical conclusion is that the relationship between the bilateral price and the quantity Montgomery indicators depends on the level of data aggregation, the choice between matched products and all available products which are to be considered, and the potential use of data filters.

The main theoretical conclusion is that the multilateral Montgomery indicators, while gaining *transitivity* and satisfying the *multi-period identity test*, lose one of the leading axioms (*identity test*). Unfortunately, it is still an open question whether it is possible to construct multilateral (and transitive at the same time) price and quantity indicators which simultaneously meet the *identity test*. According to the authors' opinion, however, even if this is not possible, multilateral indices still seem attractive from the point of view of the axiomatic approach since they meet most of the requirements. In addition, since they use the entire time window, and thus a wider range of data than their bilateral counterparts, their application to dynamically changing scanner products seems to be justified.

Appendix 1

Let us denote by $V_i^t - V_i^0$ the value change of product *i* calculated for the considered current period t and base period 0, i.e. $V_i^t - V_i^0 = p_i^t q_i^t - p_i^0 q_i^0$ and denote by ${}^{j}IP_{0,t}^{M}(\varepsilon)$ and ${}^{j}IQ_{0,t}^{M}(\varepsilon)$ the Montgomery price and quantity indicators concerning product *j* observed in only one of these considered periods, calculated in line with procedure (5). Let us assume that product *jt* is available only in period t and product *j*0 is available only in period 0. From the retailer's perspective, not observing product *jt* in period 0 and product *j*0 in period t, the value changes of products *jt* and *j*0 are as follows:

$$V_{jt}^t - V_{jt}^0 = p_{jt}^t q_{jt}^t \text{ and } V_{j0}^t - V_{j0}^0 = -p_{jt}^0 q_{jt}^0. \tag{A1}$$

The following lemma is the base for statement (8):

Lemma

Using the above notation yields

$$V_{jt}^t - V_{jt}^0 = \lim_{\varepsilon\to 0^+}\left({}^{jt}IP_{0,t}^{M}(\varepsilon) + {}^{jt}IQ_{0,t}^{M}(\varepsilon)\right), \tag{A2}$$

and

$$V_{j0}^t - V_{j0}^0 = \lim_{\varepsilon\to 0^+}\left({}^{j0}IP_{0,t}^{M}(\varepsilon) + {}^{j0}IQ_{0,t}^{M}(\varepsilon)\right). \tag{A3}$$

Proof. According to assumption (5), one has $p_{jt}^0 = q_{jt}^0 = p_{j0}^t = q_{j0}^t = \varepsilon$, and as a consequence obtains

$$\begin{aligned} {}^{jt}IP_{0,t}^{M}(\varepsilon) + {}^{jt}IQ_{0,t}^{M} &= \frac{\left(p_{jt}^t q_{jt}^t - \varepsilon^2\right)\left[ln\left(\frac{p_{jt}^t}{\varepsilon}\right) + ln\left(\frac{q_{jt}^t}{\varepsilon}\right)\right]}{ln\left(p_{jt}^t q_{jt}^t\right) - ln(\varepsilon^2)} = \\ &= \frac{\left(p_{jt}^t q_{jt}^t - \varepsilon^2\right)\left[ln\left(p_{jt}^t q_{jt}^t\right) - ln(\varepsilon^2)\right]}{ln\left(p_{jt}^t q_{jt}^t\right) - ln(\varepsilon^2)} = p_{jt}^t q_{jt}^t - \varepsilon^2, \end{aligned} \tag{A4}$$

and thus, from (A1) and (A4), there is an immediate proof of (A2) because

$$\lim_{\varepsilon\to 0^+}\left({}^{jt}IP_{0,t}^{M}(\varepsilon) + {}^{jt}IQ_{0,t}^{M}(\varepsilon)\right) = \lim_{\varepsilon\to 0^+}\left(p_{jt}^t q_{jt}^t - \varepsilon^2\right) = p_{jt}^t q_{jt}^t = V_{jt}^t - V_{jt}^0. \tag{A5}$$

Note that

$$ {}^{j0}IP^M_{0,t}(\varepsilon) + {}^{j0}IQ^M_{0,t} = \frac{\left(\varepsilon^2 - p^0_{j0}q^0_{j0}\right)\left[ln\left(\frac{\varepsilon^2}{p^0_{j0}}\right) + ln\left(\frac{\varepsilon^2}{q^0_{j0}}\right)\right]}{ln(\varepsilon^2) - ln\left(p^0_{j0}q^0_{j0}\right)} = $$

$$ = \frac{\left(\varepsilon^2 - p^0_{j0}q^0_{j0}\right)\left[ln(\varepsilon^2) - ln\left(p^0_{j0}q^0_{j0}\right)\right]}{ln(\varepsilon^2) - ln\left(p^0_{j0}q^0_{j0}\right)} = \varepsilon^2 - p^0_{j0}q^0_{j0}, \quad \text{(A6)} $$

and thus, from (A1) and (A6), there is an immediate proof of (A3) since

$$ \lim_{\varepsilon \to 0^+}\left({}^{j0}IP^M_{0,t}(\varepsilon) + {}^{j0}IQ^M_{0,t}(\varepsilon)\right) = \lim_{\varepsilon \to 0^+}\left(\varepsilon^2 - p^0_{j0}q^0_{j0}\right) = -p^0_{j0}q^0_{j0} = V^t_{j0} - V^0_{j0} \quad \text{(A7)} $$

holds.

Appendix 2

Proof of the value change test (sum test):

From (16) and (17):

$$ MIP^M_{0,t} + MIQ^M_{0,t} = \frac{1}{T+1}\sum_{\tau=0}^{T}\left({}_TIP^M_{\tau,t} - {}_TIP^M_{\tau,0} + {}_TIQ^M_{\tau,t} - {}_TIQ^M_{\tau,0}\right) = $$

$$ = \frac{1}{T+1}\sum_{\tau=0}^{T}\sum_{i\in G_{[0,T]}}\left\{L\left(p^t_i q^t_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^t_i}{p^\tau_i}\right) - L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^0_i}{p^\tau_i}\right) + \right. $$

$$ \left. + L\left(p^t_i q^t_i, p^\tau_i q^\tau_i\right) ln\left(\frac{q^t_i}{q^\tau_i}\right) - L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right) ln\left(\frac{q^0_i}{q^\tau_i}\right)\right\} = $$

$$ = \frac{1}{T+1}\sum_{\tau=0}^{T}\sum_{i\in G_{[0,T]}}\left\{L\left(p^t_i q^t_i, p^\tau_i q^\tau_i\right)\left[ln\left(p^t_i q^t_i\right) - ln\left(p^\tau_i q^\tau_i\right)\right] - \right. $$

$$ \left. - L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right)\left[ln\left(p^0_i q^0_i\right) - ln\left(p^\tau_i q^\tau_i\right)\right]\right\} = $$

$$ = \frac{1}{T+1}\sum_{\tau=0}^{T}\sum_{i\in G_{[0,T]}}\left(p^t_i q^t_i - p^\tau_i q^\tau_i + p^\tau_i q^\tau_i - p^0_i q^0_i\right) = $$

$$ = \sum_{i\in G_{[0,T]}}\left(p^t_i q^t_i - p^0_i q^0_i\right) = \sum_{i\in G_{[0,T]}} p^t_i q^t_i - \sum_{i\in G_{[0,T]}} p^0_i q^0_i = V^t_{G_{[0,T]}} - V^0_{G_{[0,T]}} \quad \text{(A8)} $$

Proof of the multi-period identity test:

Let us assume that there is the following relationship between prices and quantities of the current and base periods: $p^0_i = p^t_i$ and $q^0_i = q^t_i$. Since the multilateral price Montgomery indicator satisfies *transitivity*, one has:

$$ MIP^M_{0,1} + MIP^M_{1,2} + \ldots MIP^M_{t-1,t} = MIP^M_{0,t} = $$

$$ = \frac{1}{T+1}\sum_{\tau=0}^{T}\sum_{i\in G_{[0,T]}}\left[L\left(p^t_i q^t_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^t_i}{p^\tau_i}\right) - L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^0_i}{p^\tau_i}\right)\right] = $$

$$ = \frac{1}{T+1}\sum_{\tau=0}^{T}\sum_{i\in G_{[0,T]}}\left[L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^0_i}{p^\tau_i}\right) - L\left(p^0_i q^0_i, p^\tau_i q^\tau_i\right) ln\left(\frac{p^0_i}{p^\tau_i}\right)\right] = 0. \quad \text{(A9)} $$

Since the equality of prices and quantities from the current and base periods entails relationship (A9), one can conclude that the Montgomery price indicator satisfies the multi-period identity test (in the additive version for indicators). The analogical conclusion can be drawn for the Montgomery quantity indicator.

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